WIRELESS ENGINEER

THE JOURNAL OF RADIO RESEARCH & PROGRESS

JANUARY 1956

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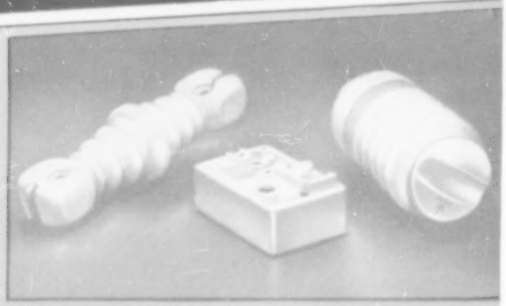

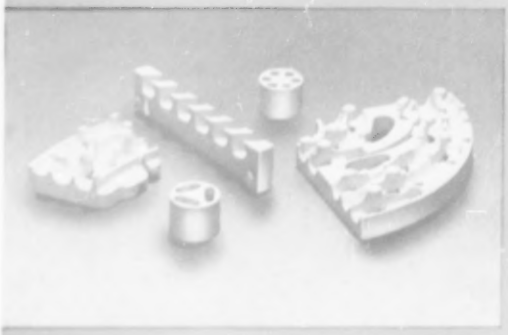

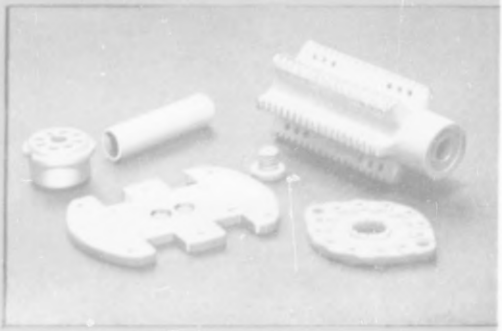

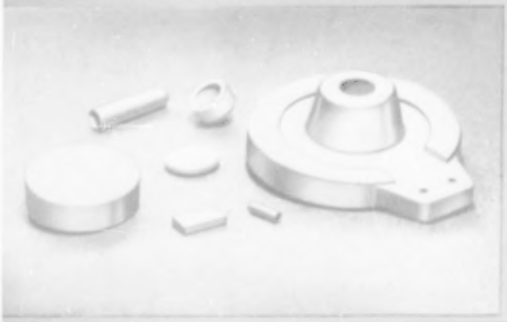

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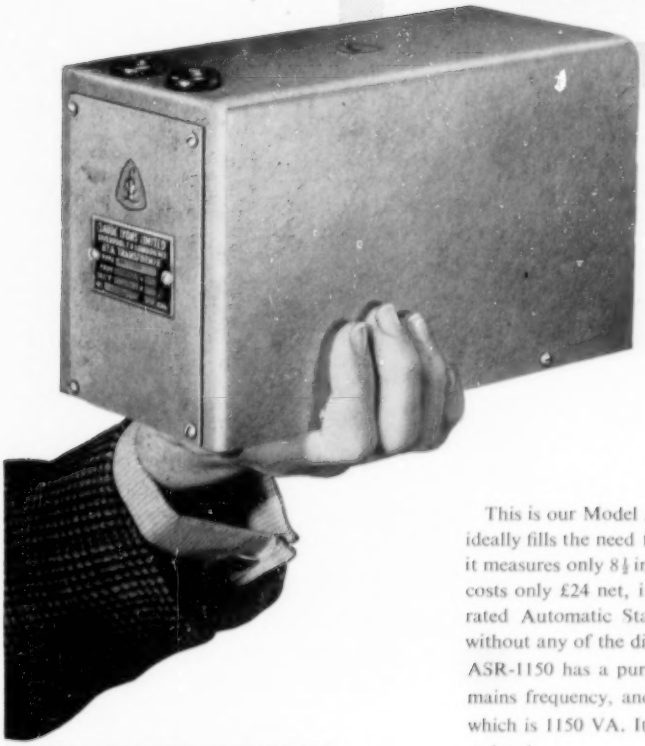

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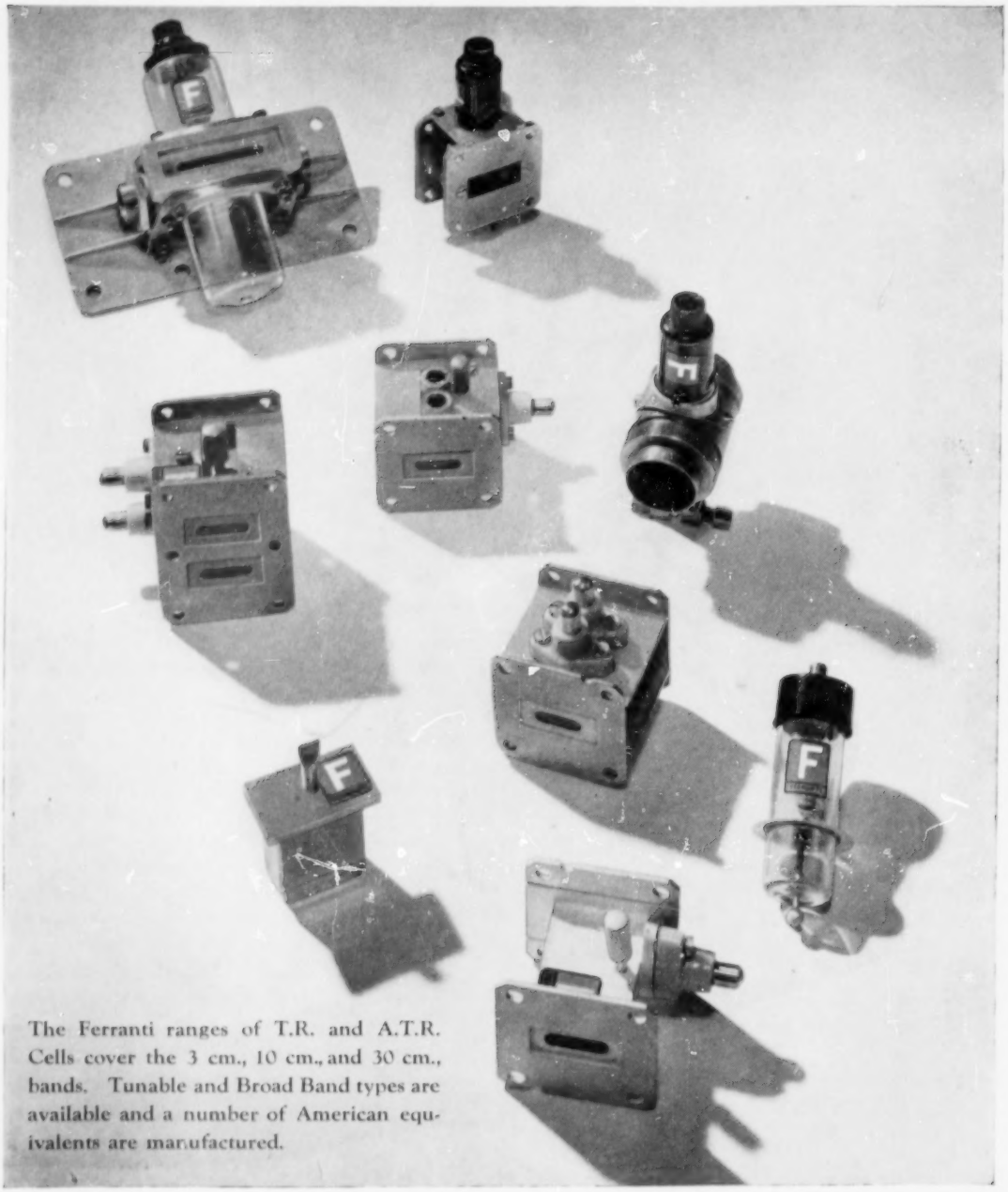

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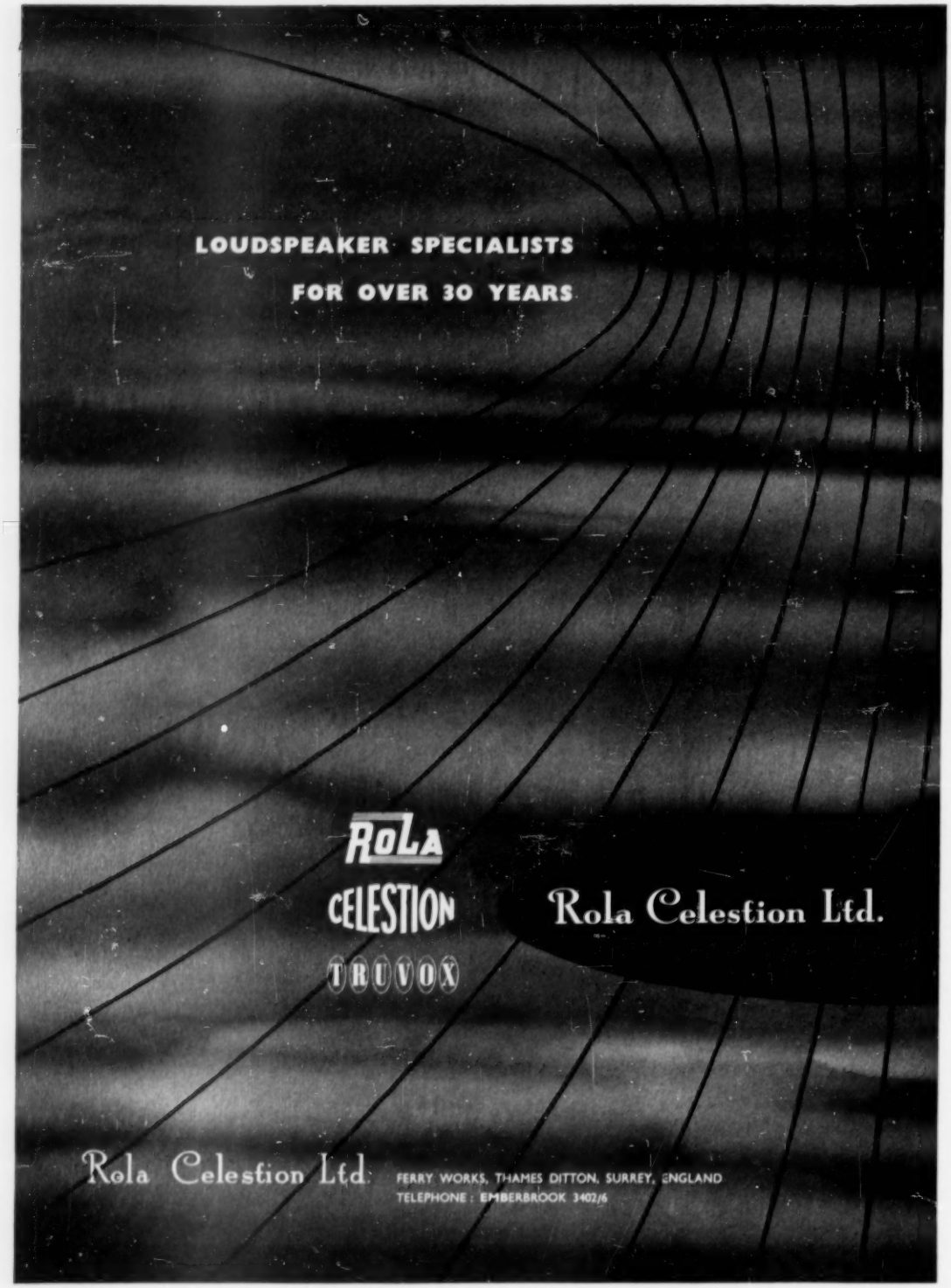
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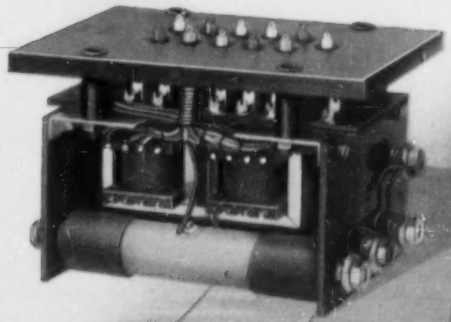

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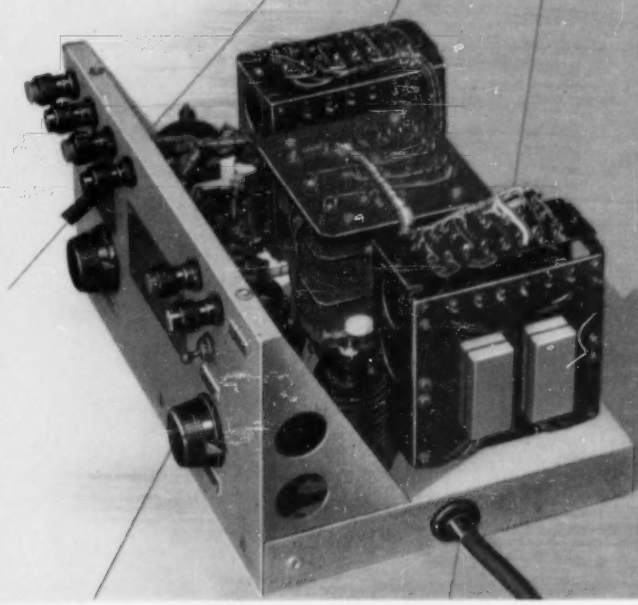

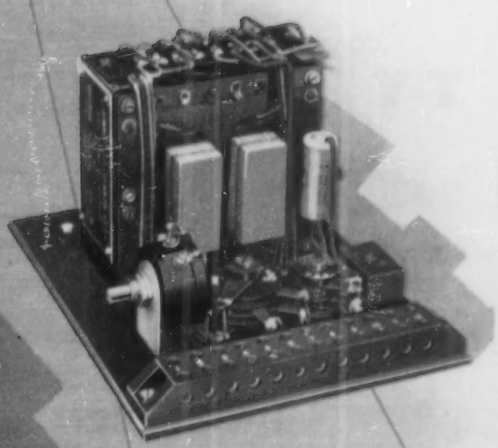

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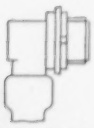

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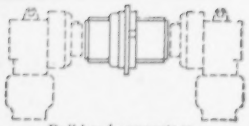

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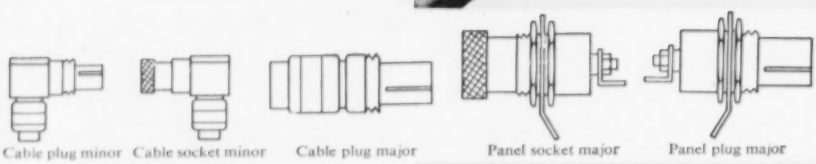

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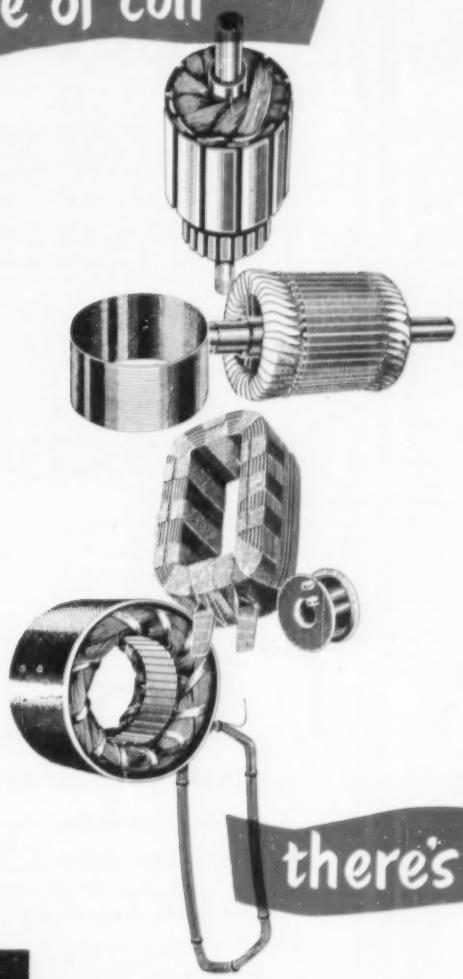

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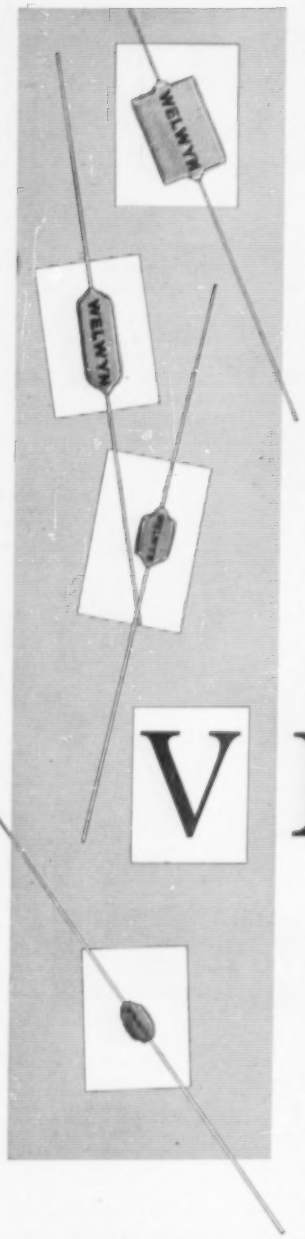

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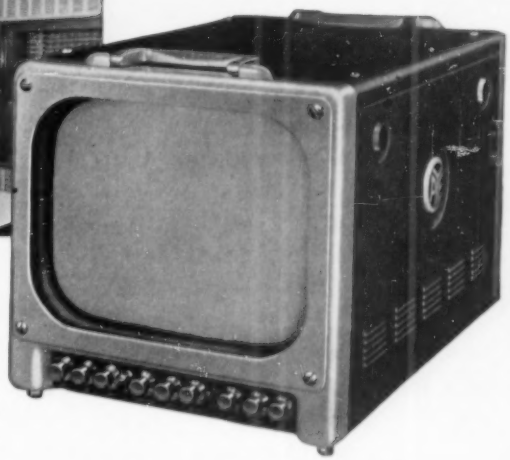

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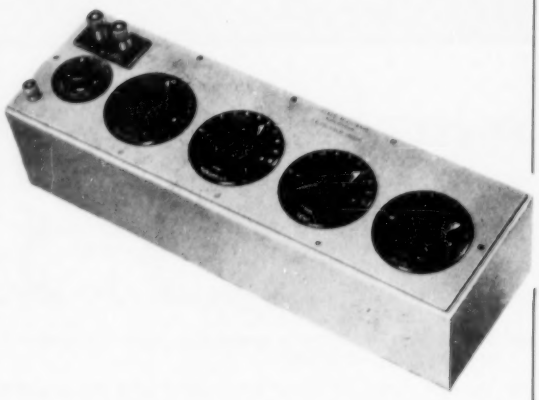

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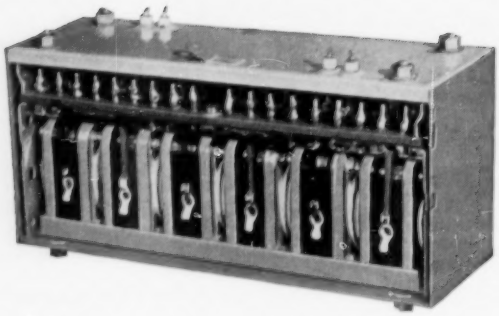

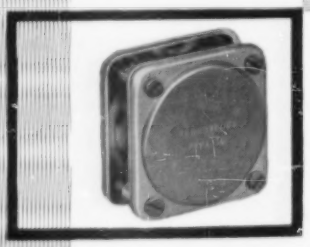

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Non-Linearity Distortion

IN television, it is a common practice to compensate for the unwanted effect of some non-linear stage by introducing an inverse non-linearity at some other point. We have seen one or two statements recently to the effect that such a process cannot be applied to audio-frequency amplifiers. We cannot see any reason why this should be so and we think the idea arises because audio engineers have the habit of thinking in terms of harmonics and combination frequencies instead of changes of waveform. A non-linear stage produces harmonics and it does seem rather incredible that a second such stage, fed with all these frequencies, should be able to produce new frequencies of precisely the right amplitudes and in the right phases for cancelling them.

We propose to examine the matter in some detail, therefore, and, to anticipate a little, it is interesting to find that in some degree this process of correcting one non-linearity by another does actually occur in every cascade amplifier. Being uncontrolled and often unsuspected its effect is rarely large, but it does result in the overall distortion of an amplifier being less than the distortion of individual stages.

Let us first of all consider an amplifier valve having the peculiar and unreal characteristic shown in Fig. 1. Over the region AB, the characteristic is quite linear and has a mutual conductance g_{m1}, say. Over the region BC it is also linear and has a higher mutual conductance, g_{m2}. The non-linearity is the abrupt change of slope at B. Let us now bias the valve to this point B and apply a signal $v \sin \omega t$ to the grid.

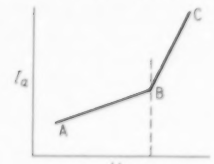

Fig. 1.

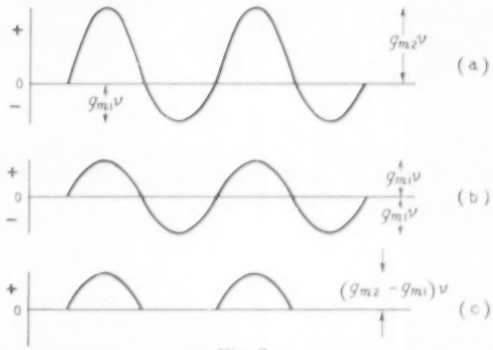

Fig. 2.

The positive excursions of the signal will all fall on BC and the negative on AB, so that the positive half-cycles of anode current will be the positive values of $g_{m2}\, v \sin \omega t$ and the negative will be the negative values of $g_{m1}\, v \sin \omega t$. The shapes of all half-cycles will be perfectly sinusoidal, but the positive ones will have a greater amplitude than the negative. The waveform is shown in Fig. 2(a) and is equivalent to the sum of (b) a sine wave $g_{m1}\, v \sin \omega t$ and (c) a rectified (half-wave) sine wave of amplitude $(g_{m2} - g_{m1})\, v$.

The Fourier series for this last is well-known to be

$$(g_{m2} - g_{m1})\, v \left[\frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{\pi}\left(\frac{1}{3} \cos 2\omega t + \frac{1}{15} \cos 4\omega t \ \ldots\right)\right]$$

and so the series for (a) is this series plus $g_{m1}\,v \sin \omega t$.

To correct the distortion of Fig. 2(a) we clearly need a further stage with a similar type of characteristic, but with the lower value of mutual conductance acting on the positive half-cycles and the higher value on the negative. Fig. 2(a) represents an anode-current waveform, however, and if we use resistance coupling to another valve, the anode-voltage waveform will be inverted. The signal fed to the grid of a succeeding valve will thus have its positive half-cycles smaller. It follows that the characteristic required for this second stage is identical with that of the valve which has caused the distortion!

If the coupling resistance has the value R_a, the positive half-cycles on the grid of the second valve are the positive values of $g_{m1}\,R_a\,v \sin \omega t$ and fall on BC (Fig. 1) where the mutual conductance is g_{m2}. The anode current is then the positive values of $g_{m1}g_{m2}R_a\,v \sin \omega t$. The negative half-cycles on the grid are the negative values of $g_{m2}R_a\,v \sin \omega t$ and fall in the region AB to produce an anode current which is the negative values of $g_{m1}\,g_{m2}\,R_a\,v \sin \omega t$. The distortion is completely cancelled.

In order that this may occur, it is essential that the bias on the second valve be correct, so that the dividing line between the positive and negative half-cycles falls at B.

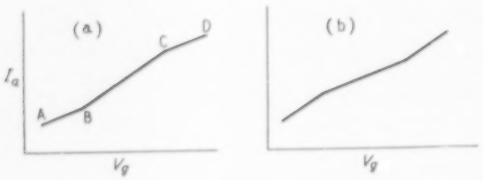

Fig. 3.

Now, if we had a valve characteristic like that of Fig. 3(a) and biased to midway between B and C, the middle section of the input sine wave would be amplified normally, but both the positive and negative peaks would be amplified less. A similar succeeding stage could now do nothing but increase the distortion further; in this case a correcting characteristic like that of Fig. 3(b) is wanted.

The form of characteristic shown in Fig. 1 is of a type which produces even-order harmonic distortion while that shown in Fig. 3 develops odd-order harmonics. Practical valve characteristics follow these forms but are, of course, curved throughout and they usually exhibit a combination of the two types. Generally, however, the form of Fig. 1 predominates, especially with triodes, but also with pentodes as long as the load impedance is not too high.

We may expect to find, therefore, that in a cascade of valve stages, there is a natural tendency for the even-order harmonic distortion to be reduced below that given by any individual stage. However, since the overall characteristic of two square-law stages in cascade tends to be S-shaped because of the reversal of phase in each stage, the odd-order distortion may be greater than that of any individual stage.

We shall now consider a more practical case, that of two valves in cascade with a.c. coupling between them and having characteristics of the general form

$$i_a = a + b\,V + cV^2$$

For valve 1, let $a = 0{\cdot}01$, $b = 0{\cdot}0016$ and $c = 0{\cdot}00006$, so that i_a is 0·01 A for zero grid volts and cuts off at -10 V on the grid. Let us bias it to -5 V and apply a sine-wave signal of 4-V peak so that

$$V = -5 + 4 \sin \omega t$$

We then have

$$i_a = 0{\cdot}00398 + 0{\cdot}004 \sin \omega t - 0{\cdot}00048 \cos 2\,\omega t$$

The distortion is second harmonic only and is 12%.

If the coupling resistance is R_a the signal applied to the second valve is the a.c. component of $-i_aR_a$. In practice, R_a might be 10 kΩ which would make the fundamental component of the input to the second valve 40 V and we should need a valve of 10 times the grid base. The coefficients a, b and c would have to be changed so that the characteristic remained of the same shape. For simplicity of computation, we shall make R_a only 1 kΩ so that the fundamental input to the valve is only 4 V and we can use the same coefficients for it. With the same bias voltage, for this second valve

$$V = -5 - 4 \sin \omega t + 0{\cdot}48 \cos 2\,\omega t$$

and we find

$$i_a = 0{\cdot}003987 - 0{\cdot}004115 \sin \omega t - 0{\cdot}0001152 \sin 3\,\omega t + 0{\cdot}000006912 \cos 4\,\omega t$$

The second harmonic has vanished completely and in its place we have 2·8% third harmonic and 0·168% fourth. This is a considerable improvement. It is true that in our example the first stage gives no gain, but this is incidental.

In practice, to achieve this sort of result, the two valves must have characteristics of the same shape and the signals must occupy the same proportion of their grid bases. Generally speaking, it is in the output stage that most distortion occurs. The penultimate stage is relatively more linear. However, it would seem that, by suitably choosing the valve and its operating conditions to *increase* its distortion, a more linear combination for the two stages could be achieved.

This conclusion is by no means new. A good many years ago* van der Ven gave some very

* "Output Stage Distortion", by A. J. Heins van der Ven, *Wireless Engineer*, August 1939, p. 383.

relevant figures. He quoted the case of a triode output valve which introduced 5% second harmonic and 1·3% third-harmonic distortion by itself. When a penultimate stage was introduced, however, the distortion changed to 1·4% second harmonic and 2·2% third.

The practical benefits of this kind of distortion reduction are not likely to be very great in the case of a.f. amplifiers. The level of distortion aimed at is often so low that linearity correction in this way would seem to demand an impracticable degree of precision. The television case is quite different, for relatively large amounts of non-linearity are unimportant and only an approximate cancellation of non-linearities suffices.

However, it does seem as though it would be practical to reduce even-order harmonic distortion by a factor of 2 or 3 without any difficulty and this may well be beneficial in reducing the amount of negative feedback required. There is one thing of considerable importance to watch however; there must be negligible frequency or phase distortion between the two non-linearities. These introduce a change of waveform which upsets the operation.

Through thinking in terms of frequencies rather than composite waveforms, the audio engineer does not always realize how frequency, and especially phase, distortion can affect waveshape. It is, in fact, quite possible to correct for a non-linearity of amplitude response by an appropriate change of the frequency and phase characteristics. It is, however, only possible when the signal is of fixed amplitude and non-sinusoidal waveform. The method is widely used in time-bases, but is useless for audio work. If one attempts non-linear distortion cancellation in a.f. circuits, therefore, one must be very careful to avoid frequency and phase distortion as well.

There is one further aspect of distortion cancellation that we should point out. In principle, its application is not confined to amplifiers. In principle, there seems no reason why it should not be used to correct in a receiver for distortion occurring in a transmitter or en route between the two. One might thus remove interference caused by cross-modulation at some point! The essential things are two: first, to equalize the frequency and phase characteristics so that they are substantially perfect between the place of introduction of the non-linearity distortion and the place of introduction of the correcting non-linearity and, secondly, to provide a correcting non-linearity which is the inverse of the original one.

Practically, of course, it would hardly be possible to meet these requirements with sufficient accuracy and we do not suggest that there is any present likelihood of such a scheme being useful.

W. T. C.

COMPATIBLE COLOUR-TELEVISION

Part 1—*Two Sub-Carrier System*

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SUMMARY.—A discussion of the requirements and possibilities for a compatible colour-television transmission system is given. A description of a transmission system using two sub-carriers for the additional colour information follows.

Introduction

IT is well known that colour television is based on the principle that any coloured light impression can be considered to be a proper addition of light of three suitably chosen primary colours—green, red and blue[1]. Every colour-television system is, therefore, arranged in such a way that the amount of green, red and blue light in each picture element of the scene to be televised is measured. Corresponding signals are produced and transmitted and the picture at the receiving end is obtained by the addition of three television pictures each in one of the three primary colours.

It will be clear that the transmission of a coloured picture of the same quality as a standard black-and-white picture will, therefore, need three times the information necessary to transmit a black-and-white picture.

In the past, several colour-television systems have been proposed[2, 3]. Most of these systems were, however, not 'compatible', a requirement which has to be fulfilled if an existing black-and-white television service is to be developed into a colour-television service, without making the black-and-white receivers in the hands of the public obsolete.

The requirement of compatibility in a colour-television system means that the information of a colour picture must be transmitted, within a normal television channel, in such a way that

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existing black-and-white receivers can reproduce the transmitted picture in monochrome, without any modification in the receiver.

The compatibility requirement restricts considerably the choice for possible colour-television systems. The N.T.S.C. system already officially adopted in the U.S.A. is an example of such a compatible system*.

Consequences of Compatibility

One of the first consequences of the compatibility requirement is that the colour-television signal has to resemble the standard black-and-white signal as much as possible. This means that the scanning rate of the colour system has to be the same or practically the same. In our opinion, a sequential transmission of the three colour signals requires an increase in the line or frame scanning rates in order to obtain a satisfying colour image. It follows, therefore, that the three signals must be transmitted simultaneously.

The second consequence of the compatibility requirement is that it will be desirable to make one of the three signals the same as the signal which would be transmitted in black-and-white television. It will be a linear combination of the primary signals. To this signal, which is usually called the 'luminance signal', other signals called 'colour signals' have to be added in such a way that they are practically invisible in normal black-and-white reception but can be separated in a colour receiver.

The inclusion of these two signals and the ordinary black-and-white signal within a normal television channel is not a very easy matter. It can only be achieved by taking advantage of certain limitations in the human visual perception.

In the first place, there exists a certain insensitivity of the eye to errors in the colour of picture detail. When the right luminance is present in the detail, such errors are hardly noticeable. Thanks to this insensitivity of the eye, the transmitted data about picture detail can be restricted to the luminance signal only. This luminance signal has, therefore, to be transmitted with the normal bandwidth, whereas the bandwidth of the other signals can be reduced considerably [4, 5, 6, 7].

Notwithstanding this decrease in bandwidth, which simplifies the problem, the sum of the bandwidths of the individual signals will still exceed the channel bandwidth. In the transmission path, therefore, the spectra of the three signals have to overlap and, at the receiver, the signals cannot be separated without mutual cross-talk. The visual effect, however, of the cross-talk can be eliminated to a large extent by making use of another limitation of the eye, viz., the well-known integration effect. As a result of this effect it is possible to make unwanted signal components of relatively low visibility. This can be done by establishing a special frequency and phase relation between the unwanted signal frequencies and the scanning frequencies so that the interfering brightness modulation in the picture has a fine structure and is of opposite sign in successive scannings[8, 9, 10, 11].

For instance, an odd multiple of half the line frequency in the higher region of the video band will be barely visible, because such a frequency shows opposite phase in successive pictures and the successive patterns in successive pictures tend to cancel each other. This holds, also, when such a frequency is amplitude modulated by a video signal.

Experiments have shown that this principle makes it possible to decrease the visual effect of the mutual cross-talk to such a level that the black-and-white reception, as well as the colour reception, of the colour-television transmission is satisfying.

Taking into account all these things, the conclusion is that a colour-television transmission can be established as follows:—

One signal is transmitted in the normal way with standard bandwidth. This signal is a linear combination of the primary signals, and is as nearly as possible the same as the signal which would be transmitted in a monochrome system. This signal is called the luminance signal.

The other signals have relatively small bandwidths—about a quarter of the channel bandwidth. They modulate sub-carriers and these modulated sub-carriers are added to the luminance signal. These signals are called the colour-signals and are, in general, also linear combinations of the primary signals. The sub-carriers are situated within the band of the luminance signal and have special frequency and phase relations with the scanning frequencies, making the presence of these modulated sub-carriers almost unnoticeable in the picture reproduced by the luminance signal.

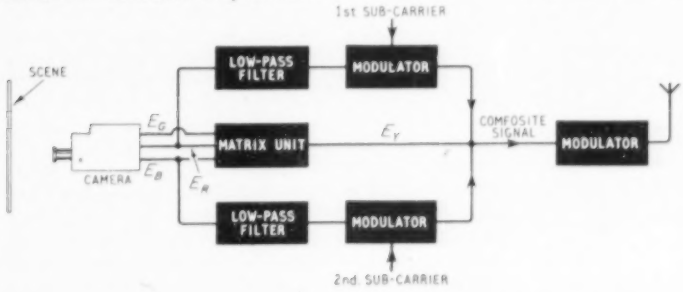
At the colour receiver, the primary signals can be obtained by demodulating the sub-carriers and by combining the demodulation results with the luminance signal. The mutual cross-talk consists of signal components which cancel out in successive lines, frames or pictures.

The colour television system accepted in the

*In the sense in which it is defined here, compatibility is not a generally-accepted requirement of a colour-television system. It is in the U.S.A., but elsewhere it has been suggested that compatibility requires only that a monochrome receiver shall, without alteration, be able to reproduce a monochrome picture from a colour transmission. In this case, all the information content of a colour transmission need not lie within the bandwidth of the monochrome receiver, but only that part of the total information which is necessary for the reproduction of a monochrome picture. Such a system might be called one of reduced compatibility and the principles of the N.T.S.C. system, as well as those of the one described in this article, can be used in it as well as in a fully compatible system.

U.S.A.—the so-called N.T.S.C. system—is based on these principles. In this system, the two sub-carriers have the same frequency and differ only in phase; in other words, only one sub-carrier is employed and it is modulated in amplitude as well as in phase[12, 13, 14].

In the rest of this article, however, we shall describe in detail a transmission system with sub-carriers of different frequencies.



$$E_y = \alpha_0 E_G + \beta_0 E_R + \gamma_0 E_B \quad .. \quad (1)$$

If α_0, β_0 and γ_0 are chosen in such a way, that

$$\alpha_0 + \beta_0 + \gamma_0 = 1,$$

the luminance signal will also vary between 0 and 1.

The luminance signal is transmitted in the normal way with normal bandwith.

Apart from this signal two other signals are transmitted with the aid of two sub-carriers, each modulated by one of these signals. The signals are chosen to be the red and blue signals.

As already mentioned in the introduction, it is possible to reduce the bandwidths of these signals, based on the physiological effect that the eye is less sensitive to colour contrast in picture details. Hence, before the sub-carriers are modulated the red and blue signals are fed to low-pass filters which remove the signal components

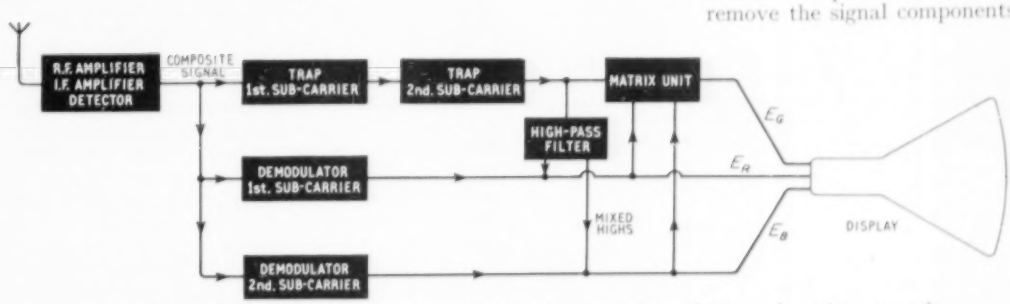


Fig. 1. *Block diagram of a colour-television transmission system, in which two sub-carriers are used.*

Formation of the Transmission Signal

In Fig. 1 is shown a block diagram of the colour-television system to be discussed. Starting at the scene to be transmitted in colour, the first component of the system is a camera, suitable for generating three primary signals corresponding to the green, red and blue light components of the scene. These signals are proportional to the amounts of green, red and blue light present in the element of the image which is scanned. These signals are indicated by E_G, E_R and E_B. They are supposed to vary between 0 and 1.

These primary signals are fed to a device (so-called matrix unit) which delivers at its output a certain linear combination of E_G, E_R and E_B. The individual contributions of the three signals to this output signal correspond to the sensitivity of the human eye to green, red and blue light respectively. The signal is called the luminance signal and will be indicated by E_y.
It can be written as:

above a certain frequency. In our experiments on a colour system matched to the existing black-and-white television system for 625 lines and 50 frames, the bandwidth was chosen to be about 2 Mc/s for the red signal and 1 Mc/s for the blue signal.

After filtering, the red and blue signals modulate sub-carriers. These sub-carriers fall within the normal video band (0–5·5 Mc/s) and have a special frequency and phase relation with the scanning frequencies (to be discussed later). Their frequencies are chosen as high as possible; however, a frequency difference of about 1 Mc/s must exist between the higher sub-carrier frequency and the sound carrier, in order to make the separation of the sub-carriers possible with obtainable receiver filter characteristics.

After modulation the sub-carriers are added to the luminance signal. In principle, this modulation may be a double-, a vestigial-, or single-sideband modulation. The signal-to-noise conditions of the colour signals at the receiver,

however, are optimum for single-sideband modulation, provided the signal amplitude is chosen to be the same in the three cases.

The combined signals form the composite video signal which is fed to a transmitter. It contains enough information to establish a satisfying colour picture at the receiver.

Receiving Process

Up to the first video amplifier, the receiver has the same circuitry as existing black-and-white receivers. The second detector delivers a signal, which can be supposed to be the same signal as the video signal present at the transmitter.

In the colour receiver, this signal is fed to two demodulators, which are intended to demodulate the sub-carriers, and to a device where the sub-carriers in the luminance signal are suppressed to a certain extent.

Both demodulators consist of a band-pass filter tuned to the appropriate sub-carrier and a detector circuit to reproduce the modulation envelope of the applied signal. Thus, at the outputs of the demodulators the red and blue signals with small bandwidth are recovered. In order to increase the sharpness of the images reproduced by these signals, the high frequencies of the luminance signal (mixed highs) are added with proper amplitude and proper time delay. These frequencies are obtained by means of simple high-pass filters.

An equivalent green signal, composed of the lower frequencies of the signal E_G and the higher frequencies of E_y, is formed in a matrix unit, where demodulator output signals with mixed highs and the luminance signal are combined.

The three signals obtained are fed to a display, suitable for reproducing a colour picture. This display may contain three projection tubes and dichroic mirrors, or may be some type of direct-viewing colour tube[15, 16, 17, 18]. When the system is well adjusted, large areas in the picture will appear in the original colour, whereas the picture detail is reproduced in black and white.

In this system the two colour signals can be separated from the composite signal without the need for synchronized detection. So sub-carrier generating and synchronizing circuits, as well as synchronized detectors, can be omitted from the receiver.

Also a synchronizing signal for the sub-carrier, which is necessary in the transmission signal of the N.T.S.C. system (the so-called 'burst') is not needed. However, such a burst may be introduced in the underlying system for other reasons. As only the sub-carrier amplitudes carry the colour information, the right amplitude ratio between the luminance signal and the sub-carriers must be maintained and this ratio must not be affected by differences in propagation for the various parts of the frequency band, or mistuning of the receiver.

In order to maintain a correct amplitude ratio in the receivers, the presence of a fixed amount of the two sub-carriers at certain predetermined moments (like the burst) may be a suitable means because it makes 'automatic ratio control' possible.

In brief, the system has now been explained. The basic idea of the system (viz., the use of two sub-carriers within the band of a normal video signal and modulated by primary signals) is not new[19, 20]. But so far as the authors know, the inherent problems of such a system have never been studied in a practical manner and solutions for these problems have never been published.

One of the major problems to be solved in the utilization of two sub-carriers is the choice of the frequency and phase of the sub-carriers in such a way that the presence of these carriers can be tolerated in black-and-white reception.

Choice of the Sub-Carriers

It has often been shown that a frequency equal to an odd multiple of half the line frequency of the television signal is much less visible in a television picture than a frequency which has an arbitrary relation to the scanning frequency. Therefore it seems obviously desirable to use sub-carriers whose frequencies are synchronized to the line frequency and are equal to odd multiples of half the line frequency. However, when two such sine waves are added to a video signal, the reproduced picture is much more disturbed than by each sine wave individually and the picture quality is seriously affected.

This effect is caused by the difference frequency of the two sub-carriers which becomes very visible because it is a multiple of the line frequency. This frequency will always appear in some degree, due to the non-linear characteristic of the cathode-ray tube.

The visual effect of this frequency turns out to be at least as important as the perceptibility of the sub-carriers themselves. This was found during experiments, carried out to determine the most satisfying frequency and phase properties of the sub-carriers. Several possibilities were investigated. Sub-carrier frequencies equal to n, $n + \frac{1}{2}$, or $n + \frac{1}{4}$ times the line frequency were applied and phase resetting at the beginning of each frame or picture period (involving phase shifts of $\pm 90°$ or $\pm 180°$) was introduced. Each pair of sub-carriers was judged by three disturbing effects: the perceptibility of a dot-structure, the stroboscopic effect of moving

dot-structures and the moiré effects caused by interference of dot-structure and picture detail.

At last one combination of two sub-carriers was found which presented a satisfactory solution of the problem and this was used in all further experiments with the transmission system. The lower sub-carrier frequency is made equal to an odd multiple of half the line frequency. In addition, the sub-carrier is given a sudden phase shift at the beginning of each frame of + and − 90° alternately. The higher sub-carrier frequency is made equal to a multiple of the line-frequency and this sub-carrier is given a phase shift of + or − 180° at the beginning of each frame.

The first sub-carrier can be written as:

$$A_1 \cos\left\{(n + \tfrac{1}{2})\, 2\pi f_1 t + \phi_1 \pm \frac{\pi}{4}\right\} \quad .. \qquad (3)$$

where the + and − sign holds alternately in succeeding frames, f_1 is the line frequency and n is an integer.

The second sub-carrier can be written as:

$$A_2 \cos\left\{m\, 2\pi f_1 t + \phi_2 \pm \frac{\pi}{2}\right\} \quad .. \quad .. \qquad (4)$$

in which m is an integer.

In consequence, the frequency difference can be represented by:

$$A_3 \cos\left\{m\, 2\pi f_1 t + \phi_2 \pm \frac{\pi}{2} - (n + \tfrac{1}{2})\, 2\pi f_1 t - \phi_1 \pm \frac{\pi}{4}\right\}$$

$$= A_3 \cos\left\{(q + \tfrac{1}{2})\, 2\pi f_1 t + \phi_3 \pm \frac{\pi}{4}\right\}$$

in which q is an integer.

This expression is of the same nature as the expression for the first sub-carrier. Hence the frequency difference between the two sub-carriers shows the same frequency and phase properties, as the first sub-carrier.

Dot Patterns of Sub-Carriers

The visual effect of the presence of the two sub-carriers can be described more or less by means of the dot patterns of the sub-carrier frequencies and their frequency difference.

The pattern of the first sub-carrier is represented in Fig. 2(a)[11]. The maxima in the brightness modulation along a line (appearing in the picture as bright dots) are indicated by:

A for the odd lines of a certain picture,
B for the even lines of that picture,
C for the odd lines of the succeeding picture,
D for the even lines of the succeeding picture.

Looking for possible stroboscopic effects, it will be noticed, that there is no direction in which the dots A, B, C, and D appear successively on a straight line. Therefore, in this pattern the effects can only be of minor importance. Moreover, the existing stroboscopic effects caused by the succession of A and C, and B and D in both vertical and horizontal directions are all equivalent, so that there exists no direction of preference for the perceptibility of stroboscopic effects.

Moving-dot structures are therefore less perceptible than those occurring in the pattern of an odd multiple of half the line frequency without sudden phase-shifts [Fig. 2(b)], where especially the succession of A, B, C, and D upwards gives rise to a rather striking stroboscopic effect.

With respect to the moiré effect caused by interference between the dot pattern and picture detail, no appreciable difference was found between the patterns of Figs. 2(a) and (b).

The pattern of the second sub-carrier is similarly represented in Fig. 2(c). In this pattern, the dots of the odd frames (A and C) coincide and so do the dots of the even frames (B and D). Stroboscopic effects are present in horizontal and vertical directions. Just as in (b) there is no direction of preference. In this case, however, the perceptibility of stroboscopic effects has not been decreased by the introduction of the phase resetting. The pattern of the multiple of the line frequency, without the phase shifts at the beginning of each frame (d), shows stroboscopic effects of the same magnitude. Yet the pattern of (c) is much less disturbing, because it gives a finer impression and less interference with picture detail, thanks to the interlacing of the dots of the two frames (A and B).

As already shown, the difference frequency shows the same pattern as the first sub-carrier. The patterns differ only in coarseness.

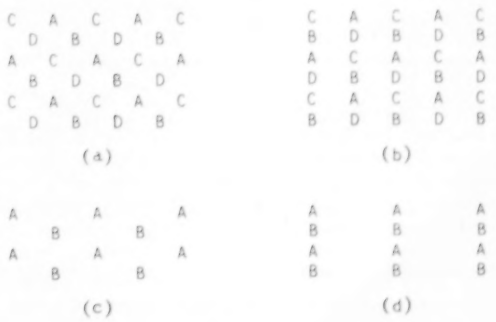


Fig. 2. *The dot pattern of the first sub-carrier with a frequency which is a multiple of half the line frequency is shown in (a) when there is a phase shift of ± 90° in alternate frames and in (b) with no phase shift. The dot pattern of the second sub-carrier with a frequency which is a multiple of the line frequency has the form shown in (c) when there is a phase shift of ± 180° in alternate frames in (d) when there is no phase shift.*

Generation of the Sub-Carriers

As already stated, the particular frequency and phase relations for the sub-carriers were used in all the experimental work. Fig. 3 illustrates the way in which they were produced.

In both generators there is an oscillator locked to the line frequency. The oscillator frequencies are $(n + \frac{1}{2})f_1$ and mf_1. The locking to the line frequency can be established in several ways, for instance, by means of a divider chain and a phase discriminator. The output of the phase discriminator depends upon the phase relation between the two frequencies and controls the oscillator frequency.

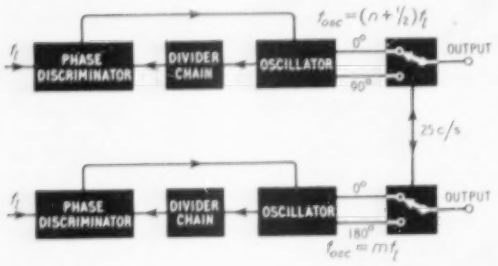


Fig. 3 (above). *Block diagram showing the method of generating the two sub-carriers.*

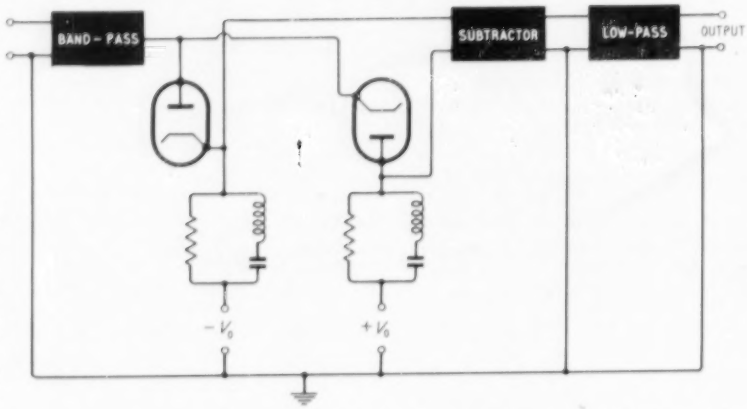


Fig. 4 (right). *Special detection circuit for the sub-carrier demodulation.*

Each oscillator delivers two sine waves, only differing in phase. This phase difference is 90° for the lower and 180° for the higher sub-carrier. The generator output is connected to the oscillator output by an electronic switch, which is switched at the beginning of each frame period. In this way the desired frequency and phase relations of the sub-carriers can be obtained.

Mutual Cross-talk

In any multiplex television system, where signals with overlapping frequency spectra are present, difficulties of cross-talk between the signals involved may arise.

The modulation of the two sub-carriers by the red and blue signals, the transmission of the composite signal modulated on a picture carrier, and the receiving and detection of this picture carrier involve no serious problems, certainly none differing from those occurring in normal television technique. There are no special problems before the stage where the received signal has to be split into three colour signals. These three signals must be obtained in such a way that the mutual cross-talk, which is caused by the overlapping of the three signal spectra, does not affect the picture quality.

First, consider the signal components of the luminance signal present in the band of a modulated sub-carrier, and suppose these signal components can be described on a certain line of a certain frame as:

$$\sum_{n=1}^{\infty} a_n \cos(\omega_n t + \phi_n) \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

where $t = 0$ indicates the beginning of the line.

If the sub-carrier is detected and the disturbing signal components are interpreted by the detector as sideband frequencies of the sub-carrier $\cos(\omega_s t + \phi_s)$, the detector output will contain a disturbing signal proportional to

$$\sum_{1}^{\infty} a_n \cos\{(\omega_s - \omega_n)t + (\phi_s - \phi_n)\} \ldots \quad (7)$$

As, in general, there is only a very small variation in the picture content from line to line and from one picture to the following picture, the luminance signal on adjacent lines and on successive scans on the same line will almost be the same.

Therefore, the disturbing signal in the detector output will only differ on adjacent lines and successive scans due to the differences in phase ϕ_s of the sub-carrier frequency.

It follows from the expression (7) that the signal components will vary in phase in the same way as the sub-carrier phase.

So cross-talk of the luminance signal in the demodulated sub-carrier consists of signal components, showing within a small region of time and space, the same phase alternation as the sub-carrier does in the luminance signal and will therefore be of reduced visibility and annoyance.

Similarly, it can also be proved, that the disturbance of the detected sub-carrier, caused by the presence of the other sub-carrier, shows phase alternation on adjacent lines and successive scans. In this case the phase alternations are equal to those which the frequency difference of the two sub-carriers shows in the luminance signal.

Thanks to the phase alternation of the cross-talk in the demodulated sub-carrier, the disturbance will be of low visibility in the corresponding picture and can be tolerated without harm to the picture quality.

However, in this reasoning it has been assumed that the interfering signal components in the band of the modulated sub-carrier are interpreted by the detector as sidebands of the sub-carrier. It will be plain that this condition may not be satisfied in some part of the picture. For instance, if in some region the sub-carrier amplitude is very small, or zero, the cross-talk in the output of the detector circuit will be the same on successive pictures and adjacent lines. In general, the undesired signal components will appear in the picture partly as a phase-alternating cross-talk and partly as a cross-talk which is independent of sub-carrier phase.

The presence of the latter type of cross-talk depends on the degree of modulation and the relative peak value of the sub-carrier. To assure its complete absence, the sub-carrier amplitude should never fall below a certain level, fixed by the peak amplitude of the undesired signal before detection.

However, for the sake of compatibility the peak value of the sub-carrier has to be kept as small as possible and it is very desirable that the sub-carrier amplitude should be zero in dark areas (100% modulation depth). We have, therefore, tried to avoid the need for a low modulation depth by applying another method of making cross-talk of low visibility. Experiments have shown this to be largely possible by the use of a special detection circuit [11]. See Fig. 4.

In this circuit, the received video signal is passed through a suitable bandpass filter and is then detected by two detectors, one following the upper envelope of this signal, the other following the lower envelope. The output signals of the two detectors are combined and the result shows a considerable reduction of the 'steady cross-talk'.

In principle, the steady cross-talk cannot be fully eliminated, because the elimination depends on the shape of the undesired signal. We can state, however, that with this detection circuit it is possible to obtain a satisfying picture from the sub-carrier information under usable conditions of peak value and bandwidth of the sub-carriers. These conditions will be specified in the second part of this article. In that part we shall give a comparison of the two sub-carrier system with the American N.T.S.C. system in order to give an impression of the advantages and disadvantages of using two sub-carriers instead of only one.

REFERENCES

[1] P. J. Bouma, "Physical Aspects of Colour", Philips' Technical Library, Eindhoven, Netherlands.

[2] P. C. Goldmark (and others), "Color Television", *Proc. Inst. Radio Engrs*, April 1942, Vol. 30, p. 162; Sept. 1943, Vol. 31, p. 465.

[3] R. D. Kell, "An Experimental Simultaneous Colour Television System", *Proc. Inst. Radio Engrs*, Sept. 1947, Vol. 35, p. 861.

[4] A. N. Goldsmith, U.S. patent No. 2,335,180, Nov. 23, 1943.

[5] A. V. Bedford, "Mixed Highs in Color Television", *Proc. Inst. Radio Engrs*, Sept. 1950, Vol. 38, p. 1003.

[6] M. W. Baldwin, "Subjective Sharpness of Additive Color Pictures", *Proc. Inst. Radio Engrs*, Oct. 1951, Vol. 39, p. 1173.

[7] K. McIlwain, "Requisite Color Bandwidth for Simultaneous Color Television Systems", *Proc. Inst. Radio Engrs*, Aug. 1952, Vol. 40, p. 909.

[8] F. Gray, U.S. patent 1,769,920, 1930.

[9] P. Merz and F. Gray, "A Theory of Scanning", *Bell. Syst. Tech. J.*, July 1934, Vol. 13, p. 464.

[10] W. Boothroyd, "Dot Systems for Color Television", *Electronics*, Dec. 1949, Vol. 22, p. 88; Jan. 1950, Vol. 23, p. 96.

[11] J. Haantjes and K. Teer, "Multiplex Television Transmission", *Wireless Engineer*, Sept. and Oct. 1954, Vol. 31, pp. 225 and 266.

[12] "Plans for Compatible Color Television", *Electronics*, Aug. 1951, Vol. 24, p. 90.

[13] D. Fink, "Alternative Approaches to Color Television", *Proc. Inst. Radio Engrs*, Oct. 1951, Vol. 39, p. 1124.

[14] Ch. J. Hirsch, "A Review of Recent Work in Color Television" "Advances in Electronics", 1953, Vol. V, Part VIII, p. 316.

[15] H. B. Law, "A Three Gun Shadow Mask Color Kinescope", *Proc. Inst. Radio Engrs*, Oct. 1951, Vol. 39, p. 1186.

[16] R. Dressler, "The P.D.F.-Chromatron", *Proc. Inst. Radio Engrs*, July 1953, Vol. 41, p. 851.

[17] A. G. van Doorn and F. W. de Vrijer, "Display Methods for Color Television", *Tijdschrift van het Nederlands Radio Genootschap*, Jan. 1955, Vol. 20, p. 35.

[18] H. R. Seelen (and others), "Development of a 21 inch Metal Envelope Color Kinescope", *R.C.A. Review*, March 1955, Vol. XVI, p.122.

[19] R. B. Dome, "Frequency Interleave Color Television", *Electronics*, Sept. 1950, Vol. 23, p. 70.

[20] R. B. Dome, "Spectrum Utilization in Color Television", *Proc. Inst. Radio Engrs*, Oct. 1951, Vol. 39, p. 1323.

(To be continued)

ELECTRON TRAJECTORIES IN COAXIAL DIODES

With Combined R.F. and Steady Fields

By R. Dehn, B.Sc., A.R.C.S., A.Inst.P.

(Communication from the Staff of the Research Laboratories of The General Electric Company, Limited, Wembley, England)

1. Introduction

THE equation of motion of an electron between coaxial cylinders is given by:—

$$\frac{d^2r}{dt^2} = \frac{e}{m} \frac{V}{r \log a/b}$$

considering only the radial component.

Here V is the potential difference between the cylinders, the cathode potential being taken as zero throughout, and r is the distance from the centre, a and b are the outer and inner radii respectively, and e/m is the charge/mass ratio for the electron, a negative quantity.

If V is constant this equation can be solved analytically. In most practical cases the electron transit time is short compared with the period of variation of any r.f. field applied to the diode, and V can therefore be regarded as substantially constant.

The writer is using a cylindrical diode in the form of a short coaxial transmission line, mounted in a waveguide system, as a waveform and power monitor for microwaves at frequencies of about 3,000 Mc/s. This type of diode was originally developed at the Services Electronics Research Laboratory, Baldock[1].

At the frequencies mentioned the electron transit time is of the order of several r.f. cycles, and can therefore no longer be neglected. The equation of motion has to be modified accordingly, introducing V as a time-dependent variable.

Experiment and theory show that a certain number of electrons will reach the anode and others return to the cathode under the influence of the r.f. field. Since no external p.d. is applied to the valve, and since the output impedance is of the same order as the dynamic impedance of the diode (i.e., about 100 ohms), the anode assumes a negative potential with respect to the cathode. The electron is therefore subject to a steady retarding field and to an r.f. field both acting simultaneously.

2. Theory

Very little theoretical work seems to have been published on electron trajectories in diodes involving large transit angles. A valve of different structure but designed for similar applications to the one considered in this paper has recently been described by Bronwell et al.[2] Their theoretical analysis includes the effect of space charge but considers the plane parallel case only. A brief treatment for fairly small transit angles (i.e., not greater than 2π radians) and applicable to plane parallel triodes has been given by Gavin[3].

It appeared desirable, therefore, to compute the electron trajectories for some specific cases of practical importance. Apart from the general interest of the problem such computation would allow an estimate of what proportion of electrons would reach the anode under given conditions.

In order to simplify analysis the following assumptions have been made:—

(1) The effect of space charge has been neglected.

(2) The initial or thermal velocities of the electrons have been neglected. This seems reasonable in view of the order of magnitude of these velocities compared with the anode–cathode potentials here considered.

(3) Since zero initial electron velocity has been assumed, and both r.f. and d.c. fields act radially, only radial components of motion need be considered in the analysis.

The equation of motion now becomes:—

$$\frac{d^2r}{dt^2} = \frac{e}{m} \frac{V_0 \sin(\omega t + \alpha) + V_{dc}}{r \log r_a/r_c}$$

where V_0 = peak r.f. potential applied between anode and cathode
V_{dc} = steady potential between anode and cathode
[cathode potential is again taken as zero]
r = distance from centre
r_a = anode radius
r_c = cathode radius
α = phase of entry of electron
ω = $2\pi \times$ frequency

The equation can no longer be solved in general terms so that numerical methods have to be employed for dealing with each particular set of conditions. The programme of computation was carried out by Messrs. Elliott Bros., on one of their digital computors.

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3. Results

Electron trajectories have been obtained for a few parameters which correspond closely to the conditions under which the diode is used in practice. As mentioned earlier, at a frequency of 3,000 Mc/s ($\omega = 6\pi \times 10^9$) the electron transit time is of the order of several r.f. cycles. If the electron leaves the cathode at zero velocity it must receive sufficient initial acceleration to reach the anode against both the negative d.c. field and the subsequent retarding half-periods of the r.f. field. It must be remembered that the field (for a given anode–cathode potential) is non-linear, varying inversely with r. Hence the electron experiences maximum acceleration near the cathode.

A preliminary consideration suggested that an electron emitted early enough during the positive half cycle would reach the anode. As the phase of entry is increased a point would be reached when the initial acceleration proved insufficient, and the electron would return to the cathode during the negative half cycle. It was consequently assumed that, unless the electron thus returned to the cathode during the first cycle it would invariably reach the anode. Detailed investigation showed, however, that this conclusion was incorrect. The electron may move towards the anode for several r.f. cycles and then gradually reverse and finally return to the cathode.

Systematic variation of the phase of entry showed the existence of several critical angles. Fig. 1 gives the actual trajectories obtained for a particular set of conditions:—

$$V_0 = 4{,}000 \text{ volts}$$
$$V_{dc} = -300 \text{ volts}$$
$$r_a = 0{\cdot}675 \text{ cm}$$
$$r_c = 0{\cdot}045 \text{ cm}$$
$$\omega = 2\pi \times 3 \times 10^9 \text{ radians/sec}$$
(i.e., $f = 3{,}000$ Mc/s)

Three points were plotted for every r.f. cycle.

For small phase angles, $\alpha = 60°$, 90°, 105°, the electron possesses positive velocity throughout its motion and reaches the anode after between one and three r.f. cycles. Between 105° and 110° there is a critical angle such that the electron just fails to reach the anode and gradually turns back towards the cathode. The most surprising fact, which was then discovered, was that even after such a reversal of velocity the electron may still reach the anode undergoing a second reversal near the cathode. Trajectories for 110° and 113° show this quite clearly. There is a further critical angle, or small range of angles, for which this second reversal does not take place, and the electron actually returns to the cathode. This

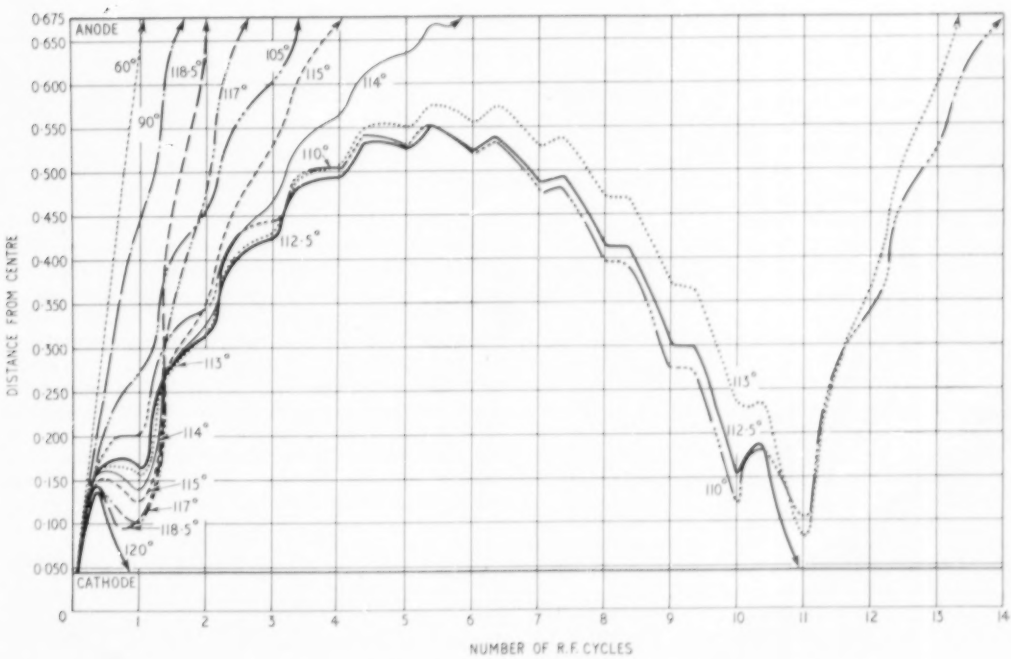

Fig. 1. *Electron trajectories for various phase angles* α.

is here shown for an angle of 112·5°. The next critical angle is reached when the electron begins to experience a reversal during the first retarding half cycle, so that the next accelerating half cycle finds the electron nearer to the cathode and therefore in a stronger field. Thus it experiences considerable acceleration and generally reaches the anode fairly quickly. This first shows at 114° for the case here considered. As the phase angle is further increased the electron approaches the cathode more and more closely, thus becoming subject to rapidly increasing accelerating fields at the commencement of the second cycle. Angles from 114° to 118·5° clearly show this in the figure. It is evident that for some of these angles (e.g., 117°, 118·5°) the electron reaches the anode more quickly than for some angles which are less than the first critical angle (e.g., 105°). Finally, near 120° a phase angle is reached such that the electron returns to the cathode during the first cycle.

These results present a remarkable picture, much more complex than the first superficial analysis had suggested. Electrons may return to the cathode either during the first negative half cycle or after a considerable number of cycles. Likewise they may reach the anode either directly after a few cycles or after a much larger number of cycles having first suffered two reversals, one near the anode and another one near the cathode.

In the case shown in Fig. 1 electrons approach the cathode following a reversal near the anode after eleven cycles. This may lead to an increase in space-charge density due to these electrons; it also suggests the possible generation of weak transit-time oscillations of approximate frequency 3,000/11 Mc/s, or, in general, some submultiple of the applied r.f.

A few additional trajectories were obtained for different conditions, first for a larger cathode ($r_c = 0{\cdot}1125$ cm) and then for $V_0 = 2{,}000$ V (small cathode). These are shown in Fig. 2. It appears that the general picture is little changed, but that the respective critical angles are reduced.

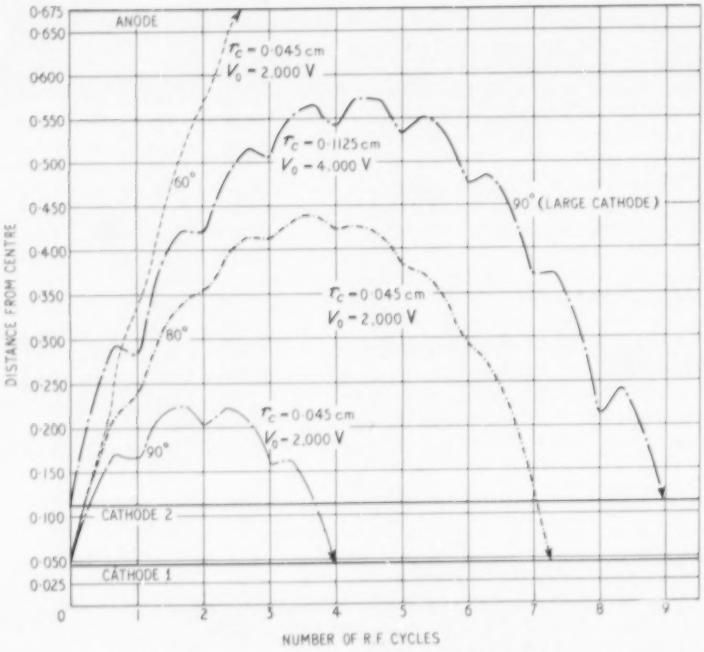

Fig. 2. *Electron trajectories for different cathode sizes and voltages.*

4. Conclusion

Experimental results suggest that, in general, about half the r.f. power going through the diode, regarded as a coaxial transmission line, is used up in cathode heating by electron bombardment. Obviously all electrons, whatever their path, must derive their energy from the r.f. field. If it is assumed that the total energies possessed respectively by the electrons which reach the anode and by those which return to the cathode are approximately equal, then it would follow that about half the total number of emitted electrons will reach the anode, the remainder being returned to the cathode. This seems in approximate agreement with the results obtained for the critical angles suggesting that electrons emitted during the first quarter cycle reach the anode and those emitted during the second quarter are returned to the cathode by the r.f. field. Obviously no electrons are emitted during the negative half cycle.

REFERENCES

[1] P. O. Hawkins et al., "A Diode Rectifier of Microwaves", *S.E.R.L. Tech. Journal*, July 1953, Vol. 3, p. 38.

[2] A. B. Bronwell et al., "Vacuum-Tube Detector and Converter for Microwaves using large Electron Transit Angles", *Proc. Inst. Radio Engrs*, July 1954, Vol. 42, p. 1117.

[3] M. R. Gavin, "Electron Pump Effect at High Frequencies", *Wireless Engineer*, February 1938, Vol. 15, p. 81.

CONSTANT-FREQUENCY OSCILLATORS

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SUMMARY.—The conditions for the frequency of a regenerative oscillator to be independent of changes in the input and output resistances of the maintaining amplifier are derived. It is shown that the input, output, and transfer impedances of the feedback network must be resistive at the oscillation frequency. This can be achieved by inserting reactances of definite specified values in the input and output leads. Expressions for the value of these reactances are derived for the usual type of *LC* oscillator with mutual-inductance coupling. The values depend greatly on whether the losses in the coils can be represented mainly by a series or by a parallel resistance. Expressions for the stability of an imperfectly-stabilized oscillator are also derived. The results are compared with those obtained by Llewellyn. It is shown that the values of stabilizing reactances specified by Llewellyn are not in general satisfactory, though in some instances they may produce a marked improvement in stability. Full experimental confirmation is given.

1. Introduction

ONE of the most important requirements of an oscillator is that its frequency should be substantially independent of moderate changes in the supply voltages. In the early 1930's constant-frequency oscillators were the subject of many papers and much controversy. Conflicting theories and experimental results were advanced by various writers, but despite prolonged discussion no satisfactory conclusions appear to have been reached. Interest waned, and although the situation has since become gradually clarified some problems remain unsolved. Of all this effort little survives in modern text-books except for references to the work of Llewellyn[1] and Groszkowski[2].

The two most important results of varying the supply voltages are:—

(a) Changes in inter-electrode capacitances and electron transit times.

(b) Variations in the functions relating changes of electrode currents to changes of voltages.

The relative importance of the different effects depends largely on the frequency of operation. At high frequencies it is possible, as Heising[3] has demonstrated, to secure overall stability by adjusting the oscillator in such a way that frequency changes due to (a) and (b) cancel one another. At low frequencies the effects of (a) are usually negligible. The effects of (b) are changes in the oscillation amplitude and harmonic content, both of which will, in general, change the oscillation frequency. Only (b) is considered further.

Variation of frequency with harmonic content has been adequately treated by Groszkowski and previous writers[4], and will not be considered here. It is therefore assumed that the electrode harmonic voltages are negligibly small. All frequency changes may then be ascribed to the variation of the effective input and output resistances of the valve caused by the change of oscillation amplitude.

This is the situation which was examined by Llewellyn, with the additional assumption that the oscillatory circuit is devoid of resistance, the only sources of dissipation being the grid leak and the grid-input resistance of the valve. Since in most practical oscillators this condition is not even approximately satisfied, a new and more general analysis of the problem seems desirable.

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2. Oscillation Frequency

Fig. 1 shows the arrangement to be analysed. The feedback network is specified by the open-circuit input impedance Z_i between 1 and 3, the open-circuit output impedance Z_o between 2 and 3, and the open-circuit transfer impedance Z_t between 1,3 and 2,3. Z_o will include the grid leak and capacitor, and Z_t the anode-circuit decoupling network, if any. A triode is depicted but the results apply equally to pentodes and tetrodes.

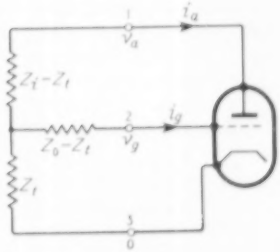

Fig. 1. *Oscillator circuit.*

It is assumed temporarily that the grid current depends only on the grid voltage v_g, and that the cathode current i_k is a function of $v_g + v_a/\mu$. In a triode the anode current $i_a = i_k - i_g$, and in a multigrid valve it is a fraction b of this. Thus in all cases

$$i_g = e(v_g),\ i_a = f(v_g + v_a/\mu) - bi_g \quad . . \quad (1)$$

where e and f are single-valued functions and μ and b are assumed to be constants. Later it will be shown that when the stability requirements are satisfied these restrictions can be removed,

and both i_a and i_g can be any single-valued functions of v_g and v_a.

The object of the analysis is to find the conditions under which the oscillation frequency is independent of μ, b, and the forms of the functions e and f.

The currents may be non-sinusoidal, but since the alternating parts of v_g and v_a have been assumed sinusoidal, only the fundamental-frequency components in i_a and i_g need be considered. Denoting such components by the suffix 1, let

$$v_{g1} = V_{g1} \cos \omega_0 t,\ i_{g1} = I_{g1} \cos \omega_0 t$$

Then $v_{g1}/i_{g1} = V_{g1}/I_{g1} = r_g$ defines the grid-input resistance of the valve.

Let $v_{g1} + v_{a1}/\mu = V_e \cos(\omega_0 t + \theta)$ where θ is the phase angle of the cathode current with respect to the grid voltage.

Then
$$\begin{aligned} i_{a1} &= I_{a1} \cos(\omega_0 t + \theta) - b i_{g1} \\ &= g(v_{g1} + v_{a1}/\mu) - b v_{g1}/r_g \quad .. \qquad (2) \end{aligned}$$

where $g = I_{a1}/V_e$ is a real positive constant.

The anode and grid currents and voltages are also related through the impedances of the feedback network. Straightforward algebra gives these relations as

$$\left.\begin{aligned} i_{a1} &= -v_{g1}/Z_t - i_{g1} Z_o/Z_t \\ v_{a1} &= v_{g1} Z_t/Z_t + (Z_t Z_o/Z_t - Z_t) i_{g1} \end{aligned}\right\} \quad (3)$$

The currents and voltages are now regarded as complex quantities, and the impedances are measured at the oscillation frequency ω_0.

Suppose now that the feedback network is modified by connecting a resistance r_g between terminals 2 and 3. The new network could be replaced by a three-element network similar to the original but with modified values for the impedances. Denoting the new elements by Z_I, Z_o, and Z_T, it is easily shown that

$$\left.\begin{aligned} Z_I &= Z_t - Z_t^2/(r_g + Z_o) \\ Z_o &= Z_o/(1 + Z_o/r_g) \quad .. \\ Z_T &= Z_t/(1 + Z_o/r_g) \quad .. \end{aligned}\right\} \quad (4)$$

Equations (3) can then be written as

$$i_{a1} = -v_{g1}/Z_T,\ v_{a1} = v_{g1} Z_I/Z_T$$

and substituting these into (2) gives

$$1 = -g(Z_T + Z_I/\mu) + b Z_T/r_g$$

Let $Z_T = R_T + jX_T$, $Z_I = R_I + jX_I$. Equating separately the real and imaginary parts of this equation,

$$g(R_T + R_I/\mu) - b R_T/r_g = -1 \quad .. \qquad (5)$$

and $g(X_T + X_I/\mu) - b X_T/r_g = 0$, from which

$$(X_T + X_I/\mu)/(R_T + R_I/\mu) = X_T/(R_T - r_g/b) \quad (6)$$

This equation determines the oscillation frequency. Equation (5) defines the oscillation amplitude, and since g is positive R_T must be negative in order to maintain the oscillation.

3. Conditions for Frequency Stability

In the frequency equation (6) the quantities pertaining to the amplifier are μ, b and r_g, and changes in the electrode potentials or the substitution of one valve by another will usually alter all of these. Hence, if the frequency is to be independent of changes in the amplifier, the solution of (6) must be independent of μ, b and r_g.

The left-hand side of (6) is independent of μ if $X_I/X_T = R_I/R_T$, and the equation then becomes

$$X_I = X_T = 0 \quad .. \quad .. \quad .. \qquad (7)$$

i.e., Z_I and Z_T are resistive at ω_0. This makes the frequency independent also of b. Condition (7), together with the fact that R_T must be negative, means that the alternating anode and grid voltages are in phase opposition. Conversely if these voltages are in phase opposition the frequency is independent of μ and b.

Equation (7) can be written in terms of the primary quantities Z_t, etc., as follows. From (4), $Z_T = r_g(R_t + jX_t)/(r_g + R_o + jX_o)$. Since Z_T is to be real the right-hand side of this expression is real. Hence

$$X_t/R_t = X_o/(r_g + R_o) \quad .. \quad .. \qquad (8)$$

Also, from (4),

$Z_I = R_t + jX_t - (R_t + jX_t)^2/(r_g + R_o + jX_o)$

On substituting for $r_g + R_o$ from (8) the condition for Z_I to be real is

$$X_t X_o = X_t^2 \quad .. \quad .. \quad .. \qquad (9)$$

(8) and (9) together are the conditions for the frequency to be independent of μ and b, and these can be satisfied in an infinite number of different ways. For example, X_o could be given any arbitrary value by the addition of a suitable reactance to the network, and (8) could be solved to find the corresponding value of ω_0. X_t could then be adjusted to satisfy (9) at this frequency.

In order that ω_0 should be independent also of r_g further restrictions must be imposed. The solution of equation (8) must be independent of r_g and this is possible only if both sides are 0. (8) and (9) then reduce to

$$X_t = X_o = X_t = 0 \quad .. \quad .. \qquad (10)$$

This is the stability criterion. If $\mu = \infty$ the value of X_t can have no influence, and the criterion reduces to $X_o = X_t = 0$. If $r_g = \infty$ (i.e., no grid current) the value of X_o is immaterial and then $X_t = X_t = 0$ is a sufficient criterion.

When the stability requirements are satisfied the restrictions of equation (1) on the forms of the anode and grid current functions can be removed. Since the frequency is independent of μ and b, these can be functions of v_g and v_a, and so i_a can be any single-valued function of v_g and v_a. Similarly, since the frequency is independent of r_g, r_g and so also i_g can be any single-valued function of v_g and v_a. The fact that v_g and v_a

are in phase opposition ensures that the grid-input impedance is resistive.

Any given feedback network can be made to satisfy condition (10) by the insertion of suitable reactances X_a and X_g in the anode and grid leads. Since these reactances cannot affect the value of X_t, the oscillation frequency is given by $X_t = 0$, and X_a and X_g are chosen to make $X_i = X_o = 0$ at this frequency. It is clear that X_a and X_g have definite fixed values. This conclusion is totally different from that of Llewellyn's theory, according to which stabilization can be achieved by a single reactance inserted in either anode or grid leads, and if two reactances are used one can have an arbitrary value.

Llewellyn assumed that the elements in the feedback network are pure reactances, the sole sources of dissipation being the grid leak and grid-input resistance of the valve. It might, therefore, appear that Llewellyn's results would be obtained as the limiting case of the general theory when R_i, R_o, and R_t tend to 0. This expectation, however, is not fulfilled, for the criterion (10) is independent of resistance values. The reason for this failure will appear later, meanwhile, to obtain Llewellyn's results, a return is made to equation (7).

If the feedback network is purely reactive then from (4)

$$Z_T = jX_t r_g/(r_g + jX_o)$$

It is obvious that (7) cannot now be satisfied for any finite values of X_o and X_t, but if both are very large compared with r_g, then

$$Z_T \approx r_g X_t/X_o + jX_t(r_g/X_o)^2 = R_T + jX_T$$

X_T will approach 0 and R_T will be finite if both X_o and X_t tend to ∞ while their ratio remains finite. Similarly,

$$\begin{aligned} Z_I &= jX_i - (jX_t)^2/(r_g + jX_o) \\ &\approx r_g(X_t/X_o)^2 + j(X_i - X_t^2/X_o) \\ &= R_I + jX_I \end{aligned}$$

So $X_I \to 0$ if $X_t X_o = X_t^2$, which is the same as condition (9). Hence the stability criterion is that (9) shall be satisfied and at the same time all three reactances shall be infinite. The stabilizing reactances X_a and X_g are not now fixed but may take any values consistent with (9).

Fig. 2 shows the arrangement analysed by Llewellyn, using the conventional equivalent circuit for the valve. It may be noted in passing that this representation is not accurate when grid current flows, but the final result is the same in this instance. Applying the criterion $X_t X_o = X_t^2$ gives

$$\begin{aligned} 2X_m(X_1 + X_m)(X_2 + X_m) \\ = (X_1 + X_a)(X_2 + X_m)^2 \\ + (X_2 + X_g)(X_1 + X_m)^2 \end{aligned}$$

which is Llewellyn's formula. The oscillation frequency is that for which $X_1 + X_2 + X_3 + 2X_m = 0$, and this makes X_i, X_o, and X_t infinite as required.

The reason why Llewellyn's results cannot be obtained by taking limits in the general theory lies in the nature of the reactance function. In the neighbourhood of an antiresonance the reactive component of any physical driving-point impedance function passes from a large positive to a large negative value, passing through zero at the antiresonant frequency. The smaller the network dissipation the larger are the values of the maximum and minimum reactances but, however small the dissipation may be, the reactance still passes through zero at the antiresonance. However, if the impedance network is assumed to be completely loss-free, the reactance rises to ∞ on one side of the antiresonance, to $-\infty$ on the other side and is indeterminate at the antiresonant point. This explains the appearance of infinities in Llewellyn's method and zeros in the more general theory.

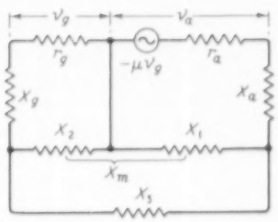

Fig. 2. *Llewellyn's circuit.*

Condition (9), which is equivalent to Llewellyn's criterion, can be regarded as the limit of the general formula provided the network dissipation is not actually zero. If this dissipation is very small compared with that in the grid-input resistance, the value of R_o is very large compared with r_g at the oscillation frequency. Then equation (8) will be satisfied for almost the same value of ω_0 whatever the value of r_g, leaving (9) as the sole criterion for stability.

If $\mu = \infty$ Llewellyn's theory indicates that no stabilizing reactances are required, for the value of X_i is then immaterial and the sole criterion is that X_t and X_o shall be infinite. The addition of any finite reactance X_g to X_o cannot therefore have any influence on the frequency.

4. Application of the Criterion

The general theory is valid for any type of harmonic oscillator, but all the important points are brought out by a consideration of the particular circuit shown in Fig. 3. X_a and X_g are the stabilizing reactances, and R_b and C_b are the grid leak and bypass capacitor. Losses in the coils L and L_1 (coupled by the mutual inductance M) are represented by two series resistances r and r_1 and the parallel resistance R. The reason for this choice will be discussed later.

Let $\omega_r^2 LC = 1$, $M = -k\sqrt{LL_1}$

$$Q_p = R/\omega_r L,\ Q_s = 1/\omega_r C r,\ Q = Q_p Q_s/(Q_p + Q_s) \quad .. \quad .. \quad .. \quad (11)$$

Elementary circuit analysis gives the input, output, and transfer impedances of the network (corresponding to those indicated in Fig. 1) as

$$\begin{aligned} Z_i &= jX_a + (L/M + r/j\omega M)Z_t \\ Z_o &= jX_g + (\omega M/\omega_r L)(\omega/\omega_r - j/Q_p)Z_t \\ &\quad + R_b/(1 + j\omega C_b R_b) + j\omega L_1 + r_1 \\ Z_t &= j\omega_r M/\{j/Q + (1 + r/R)\omega_r/\omega - \omega/\omega_r\} \end{aligned} \quad .. \quad .. \quad .. \quad (12)$$

Let ω_0 be the angular frequency of oscillation at which the three impedances are to be resistive. Z_t is resistive when

$$\omega = \omega_0 = \omega_r(1 + r/R)^{\frac{1}{2}} = \omega_r(1 + 1/Q_p Q_s)^{\frac{1}{2}} \quad .. \quad .. \quad .. \quad (13)$$

and then $Z_t = R_t = \omega_r MQ$. Using these relations, Z_i is resistive at ω_0 if

$$\begin{aligned} X_a &= Qr/(1 + r/R)^{\frac{1}{2}} \\ &= \omega_0 L/(1 + Q_s/Q_p)(1 + 1/Q_p Q_s) \end{aligned}$$

Since the product $Q_p Q_s$ will usually be very large compared with 1, X_a is the reactance of an inductance of value

$$L_a = L/(1 + Q_s/Q_p) \quad .. \quad .. \quad .. \quad (14)$$

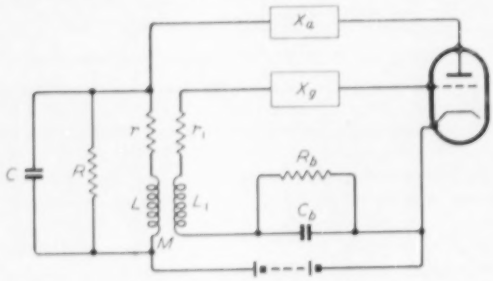

Fig. 3. *Coupled-coil oscillator.*

Similarly from (11), (12) and (13), Z_o is resistive at ω_0 when

$$\begin{aligned} X_g &= \omega_0 C_b R_b^2/\{1 + (\omega_0 C_b R_b)^2\} \\ &\quad - \omega_0 L_1(1 - k^2 Q/Q_p) \end{aligned}$$

In all practical oscillators $\omega_0 C_b R_b \gg 1$. Assuming also that $Q_p Q_s \gg 1$, X_g is the reactance of a capacitance C_g given by

$$1/C_g = \{1 - k^2/(1 + Q_p/Q_s)\}L_1/LC - 1/C_b$$

If C_b is large

$$C_g = CL/L_1\{1 - k^2/(1 + Q_p/Q_s)\} \quad .. \quad (15)$$

Alternatively C_b may be given this value in which case $C_g = \infty$; i.e., $X_g = 0$. This arrangement would be preferred whenever possible, for a separate stabilizing capacitor would require to be shunted by a large inductor to allow the direct component of grid current to pass.

It is clear that the magnitudes of the stabilizing reactances depend greatly on the relative values of r and R. It is often assumed that the losses in a coil can be represented either by a parallel or by a series resistance, and for many purposes it is immaterial which convention is chosen. In the present application, however, the distinction is vital.

Given an isolated coil there is no experimental method of distinguishing between a series and a parallel resistance but, when a second coil is coupled magnetically to the first, the phase of the p.d. across the second coil with respect to the current in the first coil is $\frac{1}{2}\pi$ if the losses can be represented entirely by a series resistance. If a parallel loss resistance is included the relative phase is less than $\frac{1}{2}\pi$; i.e., the mutual impedance between the coils then has a resistive as well as an inductive component.

For a single-layer air-cored coil with widely-spaced turns the losses are substantially the same as those in the same length of straight wire. There is no doubt that the losses are here correctly represented by a series resistance. However, when the turns are close together, when dielectric losses occur in the coil former, or when a ferromagnetic core of a screening can is used, the losses are greatly increased, and part of this increase must be represented by a parallel resistance.

The problem of how the various sources of loss should be represented is a complicated one and cannot be discussed here, but as an example the air-cored multi-layer coil used in the experiments had $Q_p = 32$ and $Q_s = 47$ at 50 kc/s. Thus 60% of the total loss is represented by the parallel resistance. All the loss in this coil occurs in the copper.

It might appear that a parallel resistance should also be placed across the second coil L_1 in Fig. 3, but this is unnecessary, for such a network could be reduced to the form of Fig. 3. The network of Fig. 3 is in fact only one of a number of possible equivalent circuits. It has the advantage of simplicity and also allows dielectric losses in the capacitor C to be included in the parallel resistance R. Methods of measuring R and r are discussed in the Appendix.

This division of losses might account in part for some of the discrepancies observed by previous workers, as Clarke[5] has pointed out. It might also explain some of the errors which arise when the effective resistance of a coil is measured by methods involving mutual inductance[6].

If $r = 0$, (14) and (15) become

$$L_a = 0 \text{ and } C_g = CL/L_1(1 - k^2) \quad .. \quad (16)$$

independent of the value of R. If $R = \infty$ the results are

$$L_a = L \text{ and } C_g = CL/L_1 \quad .. \quad .. \quad (17)$$

independent of the value of r. When the amplifier is biased to operate without grid current the sole criterion in the last case is $L_a = L$, which is the result obtained by Mallett[7].

Formulae (14) and (15) may be compared with the results obtained by Llewellyn, which give the anode stabilizing impedance as an inductance $L_a = L\{1 + (LC/L_1C_g - 1)/k^2\}$ or a capacitance $C_a = C/\{(1 - LC/L_1C_g)/k^2 - 1\}$, the value of C_g being arbitrary. In particular if $C_g = \infty$ then

$$C_a = Ck^2/(1 - k^2) \quad .. \quad .. \quad (18)$$

and if $L_a = 0$ ($C_a = \infty$) then

$$C_g = CL/L_1(1 - k^2) \quad .. \quad .. \quad (19)$$

The values of L_a and C_g given by (14) and (15) are, of course, in agreement with Llewellyn's general formula, since both satisfy condition (9). If $\mu = \infty$ the value of X_a and consequently also of X_g can have no influence on ω_0, and so stabilizing reactances should be unnecessary, as already pointed out.

5. Degree of Stabilization

The foregoing theory, although showing what conditions are necessary for stability, gives no indication of the magnitude of the frequency changes to be expected when these conditions are not exactly fulfilled. A general treatment of this problem is straightforward but tedious; useful information can be gained by considering two particular cases. In the first, r_g is supposed very large, and the variation of ω_0 with μ is calculated; in the second, μ is supposed very large and the variation of ω_0 with r_g is found.

If $r_g = \infty$, then from (4), $Z_I = Z_i$, etc., and the frequency equation (6) becomes $X_t + Xi/\mu = 0$. Let X_a' be the correct anode stabilizing impedance, and let ω_0' be the corresponding frequency. In this state $X_i = X_t = 0$. When X_a' is changed to X_a let ω_0' change to ω_0. Then X_t changes to $(\omega_0 - \omega_0')dX_t/d\omega$, and X_i to $X_a - X_a' + (\omega_0 - \omega_0')dX_t/d\omega$.

Substituting these values into the frequency equation gives

$$\omega_0 - \omega_0' = -(X_a - X_a')/(dX_t/d\omega + \mu dX_t/d\omega).$$

This assumes that $dX_a/d\omega$ is negligible compared with $dX_t/d\omega$ which is true in all practical cases. Differentiating this equation with respect to μ gives

$$\partial\omega_0/\partial\mu = \frac{(X_a - X_a')(dX_t/d\omega)}{(dX_i/d\omega + \mu dX_t/d\omega)^2}$$

If Z_i and Z_t have the same form, near to ω_0, as the impedance of an antiresonant circuit, namely

$$Z = R/\{1 + 2jQ(\omega - \omega_a)/\omega_a\},$$

then since $\omega_a \approx \omega_0$ $dX_t/d\omega = -2R_tQ_t/\omega_0$ and $dX_t/d\omega = -2R_tQ_t/\omega_0$. Hence

$$\partial\omega_0/\partial\mu = -\tfrac{1}{2}\omega_0R_tQ_t(X_a - X_a')/(R_tQ_i + \mu R_tQ_t)^2 \quad .. \quad .. \quad (20)$$

This expression is strictly valid only for very small oscillation amplitudes, for if the amplitude is large the value of μ will vary over the oscillation cycle.

In the second case $\mu = \infty$. Then the frequency equation reduces to $X_I = 0$. Let X_g' be the correct grid stabilizing impedance and ω_0' the corresponding frequency. In this state $X_o = X_t = 0$. When X_g' is changed to X_g let ω_0' change to ω_0. Now $Z_I = Z_tr_g/(r_g + Z_o)$. Hence if X_t and X_o are small

$$X_I = \{X_t - X_oR_t/(r_g + R_o)\}r_g/(r_g + R_o) = 0$$

Substituting $X_o = X_g - X_g' + (\omega_0 - \omega_0')dX_o/d\omega$, and for X_t as before into this equation gives

$$\omega_0 - \omega_0' = \frac{-R_t(X_g - X_g')}{\{R_tdX_o/d\omega - (r_g + R_o)dX_t/d\omega\}}$$

Differentiating this with respect to r_g gives

$$\partial\omega_0/\partial r_g = \frac{-R_t(X_g - X_g')(dX_t/d\omega)}{\{R_tdX_o/d\omega - (r_g + R_o)dX_t/d\omega\}^2}$$

If Z_o and Z_t have the form of antiresonant impedances near to ω_0 then

$$\partial\omega_0/\partial r_g = \frac{\frac{1}{2}\omega_0Q_t(X_g - X_g')}{\{R_oQ_o - (r_g + R_o)Q_t\}^2} \quad (21)$$

For the oscillator of Fig. 3 it is easily shown, provided $Q_pQ_s \gg 1$, that $Q_i = Q_o = Q_t = Q$, Q being given by (11), and that $R_i = \omega_0LQ$, $R_o = Q\omega_0M^2/L$, $R_t = \omega_0MQ$. If the correct anode stabilizing reactance is an inductance L_a', then for $r_g = \infty$, substitution of these values into (20) gives

$$\partial\omega_0/\partial\mu = -\tfrac{1}{2}\omega_0M(L_a - L_a')/Q^2(L + \mu M)^2 \quad (22)$$

If the correct grid stabilizing reactance is a capacitance C_g' then for $\mu = \infty$ substitution into (21) gives

$$\partial\omega_0/\partial g_g = -\tfrac{1}{2}(C_g - C_g')/C_gC_g'Q \quad .. \quad (23)$$

where $g_g = 1/r_g$ is the grid-input conductance of the valve. The values of L_a' and C_g' are given by (14) and (15).

For the unstabilized oscillator $L_a = 0$ and $C_g = \infty$, and then (22) and (23) become [using (11)]

$$\partial\omega_0/\partial\mu = \tfrac{1}{2}\omega_0(M/L)/Q_sQ(1 + \mu M/L)^2 \quad .. \quad (24)$$

$$\partial\omega_0/\partial g_g = -\tfrac{1}{2}\omega_0r(L_1/L)\{1 + (1 - k^2)Q_s/Q_p\} \quad (25)$$

Variation of r_g will usually be the more important source of frequency change. This can be minimized by making the coefficient of coupling as near to 1 as possible, and making r small.

Equation (24) may also be expressed in terms of the equivalent anode conductance $g_a = 1/r_a$ of the valve. With this convention it is easily shown for the circuit of Fig. 3 and with $r_g = \infty$ that $-\mu\omega_0MQ = \omega_0LQ + 1/g_a$. Using this

relation between μ and g_a, and also equations (11), equation (24) becomes

$$\partial\omega_0/\partial g_a = \tfrac{1}{2}\omega_0 r \qquad (26)$$

6. Experimental Results

Experiments were started with the arrangement shown in Fig. 3, but it was found that variations in inter-electrode capacitances and harmonic content were masking the results. These difficulties were overcome by inserting suitable resistors in the cathode lead, rearranging the connections of the grid leak and capacitor, and interposing a cathode-follower buffer stage between the network and the amplifier proper. A pentode amplifier was used and the gain was controlled by adjusting the screen voltage. Variations in r_g and μ were simulated by variable resistors r_g and r_a connected across the network output and input terminals (2,3 and 1,3 in Fig. 1). The effective μ of the amplifier is $r_a g_m$, where g_m is the mutual conductance. These resistors and the amplifier gain were adjusted simultaneously to give a very small oscillation amplitude.

Inductances L and L_1 in Fig. 3 were air-cored multi-layer coils wound with enamelled copper wire. Their parameters, measured in the way described in the Appendix, were at 50 kc/s.

$L = 530\,\mu\text{H}$, $L_1 = 523\,\mu\text{H}$, $M = 215\,\mu\text{H}$,
$R = 5{,}340\ \Omega$, $r = 3{\cdot}55\ \Omega$, $r_1 = 8{\cdot}00\ \Omega$,

from which

$k = 0{\cdot}408$, $Q_p = 32{\cdot}1$, $Q_s = 46{\cdot}8$

By adding resistors in parallel with the coil L the value of R could be adjusted, and by adding resistors in series r could be changed. In this way the ratio Q_p/Q_s could be varied over a wide range. The main tuning capacitor C was $0{\cdot}0190\,\mu\text{F}$, and the oscillation frequency 50 kc/s.

The experimental method was as follows: With $r_a = \infty$ the change in ω_0 due to a definite small change in r_g was measured for various values of C_g. With $r_g = \infty$ the effect of a small change in r_a was measured for different values of L_a. In all cases the frequency change was a linear function of L_a or $1/C_g$. Hence by drawing straight-line graphs the values of L_a and C_g required for exact stabilization could be found. Finally these values were checked by varying both r_a and r_g over a wide range. This procedure was repeated for various values of Q_p and Q_s with the results shown in Table 1.

The theoretical values of L_a and C_g are calculated from (14) and (15). In view of the various experimental uncertainties the agreement with the experimental values is considered to be satisfactory. The almost constant value of C_g is due to the low value of k^2 (= 0·166). Equation (15) shows that in such cases the value of Q_p/Q_s has little effect on C_g which is approximately equal to CL/L_1; i.e., $\omega_0 C_g = 1/\omega_0 L_1$. Thus C_g is the capacitance required to tune the series circuit L_1, C_g, $r_1 + r_g$ to the oscillation frequency.

TABLE 1

Q_p	Q_s	$L_a(\mu\text{H})$		$C_g(\mu\text{F} \times 10^{-4})$	
		Meas.	Calc.	Meas.	Calc.
32·1	5·85	458	448	201	198
32·1	10·8	376	397	202	201
32·1	23·6	290	305	206	207
32·1	46·8	212	216	212	213
14·5	46·8	140	125	220	221
9·1	46·8	76	86	223	223

According to Llewellyn's theory, stabilization should be obtained with either an anode capacitance of 0·00378 μF, given by (18), or with a grid capacitance of 0·0231 μF, given by (19). Llewellyn's assumption that all the dissipation is in the grid-input resistance cannot be realized in practice but, by reducing r_g to the lowest workable value for this particular oscillator, namely 112 Ω, about 80% of the total dissipation was obtained in r_g. With the values of C_a and C_g indicated above the frequency changes produced by small variations of r_a and r_g were measured for different values of r_a. The rate of change of oscillation frequency f_o ($= \omega_0/2\pi$) could then be computed to give the results of Table 2.

TABLE 2

$C_a(\mu\text{F})$	$C_g(\mu\text{F})$	$r_a(\Omega)$	Change in f_o (kc/s per mho)	
			g_a	g_g
∞	0·0231	∞	121	−13·2
		8820	121	−13·2
		4350	121	−13·2
0·00378	∞	∞	2800	−158
		8820	2270	−125
		4350	1730	−102
∞	∞	∞	−4120	−160
		8820	−4120	−212
		4350	−4120	−260

The last three rows give the results for the same oscillator without stabilizing reactances. It is seen that Llewellyn's values, though different from those required for complete stabilization produce a marked improvement when (in this particular arrangement) the reactance is in the grid circuit. Anode stabilization is much less effective. The results for $r_a = \infty$ are of particular interest, since in this case (which corresponds to $\mu = \infty$) no stabilizing reactances should be required according to Llewellyn's theory.

A value of $r_g = 112\,\Omega$ would be very unusual. Under more realistic conditions r_g would be very much larger and almost all the dissipation would be in the coil resistances. Measurements of frequency stability for the limiting case of $r_a \to \infty$ and $r_g \to \infty$ were therefore carried out, see Table 3. The frequency changes were found to be linear with respect to moderate changes in g_a and g_g away from 0.

TABLE 3

$C_a(\mu F)$	$C_g(\mu F)$	Change in f_o(kc/s/mho)			
		g_a		g_g	
		Meas.	Calc.	Meas.	Calc.
∞	0·0231	87	89	−19	−15·3
0·00378	∞	1170	1200	−205	−194

The theoretical values are calculated from (22), (23), (25) and (26) using the measured values of the coil parameters. The anode stabilizing capacitance of 0·00378 μF is treated as an inductance of $-2680\,\mu$H. In fact it has an anti-stabilizing effect. Stability with respect to changes in g_g is about the same as for the heavily-loaded oscillator. Llewellyn's value of grid-stabilizing capacitor is very near to the correct value because of the smallness of k. If k were near to 1, the discrepancy would be much larger unless Q_s/Q_p were very large. This can be seen be comparing formulae (15) and (19).

These experiments have shown that, provided the effects of harmonics and inter-electrode capacitances are eliminated, the theoretical and actual values of the stabilizing reactances and of the frequency shifts are in good agreement. In general, the values of grid and anode stabilizing reactances specified by Llewellyn will not be satisfactory.

APPENDIX

Equivalent Circuits for Coupled Coils

The behaviour of the coupled-coil network shown in Fig. 4(a) can be specified in terms of the three impedances $j\omega L_b + r_b$, $j\omega L_c + r_c$, and $j\omega L_m + r_m$ shown at (b). To obtain the equivalent circuit (c) all that is necessary is to equate these impedances to the open-circuit input, output, and transfer impedances of (c). Thus

$$j\omega L_b + r_b = R(j\omega L + r)/(j\omega L + r + R)$$
$$j\omega L_c + r_c = j\omega L_1 + r_1 + \omega^2 M^2/(j\omega L + r + R)$$
$$j\omega L_m + r_m = j\omega MR/(j\omega L + r + R)$$

These equations may be solved to find R, r, r_1, L, L_1, and M. The general expressions are cumbersome, but if it is assumed that $Q_p Q_s \gg 1$ (which will be true in all practical cases) the relations simplify to

$$L = L_b,\ L_1 = L_c,\ M = L_m$$
$$R = \omega^2 LM/r_m,\ r = r_b - r_m L/M,\ r_1 = r_c - r_m M/L,$$ from

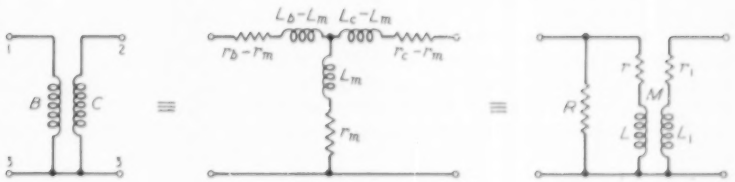

Fig. 4. *Equivalent circuits for coupled coils.*

which

$$Q_p = \omega M/r_m,\ Q_s = \omega L/(r_b - r_m L/M),\ 1 + Q_p/Q_s = r_b M/L r_m$$

L_b and r_b are found by measuring the impedance between terminals 1 and 3 in Fig. 4(a), and L_c and r_c by measuring between 2 and 3. L_m and r_m are obtained by measuring between 1 and 2, and using the previously found values of L_b, L_c, r_b, and r_c. The sign of r_m is the same as that of L_m.

REFERENCES

[1] F. B. Llewellyn, "Constant-Frequency Oscillators", *Proc. Inst. Radio Engrs*, 1931, Vol. 19, p. 2063.

[2] J. Groszkowski, "The Interdependence of Frequency Variation and Harmonic Content, and the Problem of Constant-Frequency Oscillators", *Proc. Inst. Radio Engrs*, 1933, Vol. 21, p. 958.

[3] R. A. Heising, "Stability in High-Frequency Oscillators", *Proc. Inst. Radio Engrs*, 1943, Vol. 31, p. 595.

[4] E. V. Appleton and W. H. M. Greaves, "On the Solution of the Representative Differential Equation of the Triode Oscillator", *Phil. Mag.* 1923, Vol. 45, p. 401.

[5] C. F. Clarke, "Application of the Dynatron", *Wireless Engineer*, 1934, Vol. 11, p. 75.

[6] E. B. Moullin, "Radio Frequency Measurements", 2nd Edn., p. 376. (Charles Griffin & Co. Ltd., London, 1931.)

[7] E. Mallett, "Frequency Stabilisation of Valve Oscillators", *J. Instn elect. Engrs*, 1930, Vol. 68, p. 578.

"THE ENGINEER"

Not many technical journals are yet in a position to celebrate a centenary. *The Engineer* has this distinction and, with its issue dated 4th January 1956, commemorates its first issue dated precisely 100 years earlier.

Its early days coincided with the growth of railways and it naturally devoted much space to them. Its outlook has always been broad, however, and it reported the early experiments of Marconi, although in terms which strike the modern ear as strange!

For the last 91 years the Editor-in-Chief has been a Pendred—father, son and grandson. For the present editor, therefore, the occasion is one not only of celebrating the centenary of his journal but of a long family connection with it.

ADCOCK DIRECTION FINDER

Polarization Errors Due to Aerial Bending

By W. C. Bain, M.A., B.Sc., Ph.D.

(Official communication from D.S.I.R. Radio Research Station, Slough)

SUMMARY.—It was found that the pull of the triatics (supporting the central sense aerial) on the main aerials of an Adcock h.f. direction finder caused the main aerials to bend inwards, so that a large polarization error was produced on frequencies near 20 Mc/s. Theoretical calculations are made for the case of aerials with an overall slope of 1°, with a distant loop transmitter at 6·3° elevation and with a loop tilt of 6° from the horizontal: the total polarization error found is 6° for a perfectly-reflecting ground and 19° for ground of conductivity 0·02 mho/m. In practical direction finding the errors are not likely to be as high as this on the average; but the figure of 3° is obtained for the root-mean-square error from a transmitter on 20 Mc/s at 20° elevation despite the small amount of aerial bending assumed.

1. Introduction

IN one type of U-Adcock high-frequency direction finder in common use a sense aerial of cage construction is suspended by triatics from the tops of the main aerials, which are self-supporting. The pull of the triatics varies according to the tension in the sense aerial, but can be large enough to produce appreciable inward bending in the main aerials. It is shown in this paper that this effect will lead to polarization errors, which, though negligible at the low-frequency end of the range, become large at frequencies of the order of 20 Mc/s.

2. Experimental Discovery of the Error

Polarization error of appreciable magnitude due to aerial bending was first discovered when investigations on a particular direction finder were made to determine the cause of an observed error which rose rapidly with frequency. The installation was of the type mentioned above, with main aerials 9·1 m high tapered towards the top and spaced 6·1 m apart; an earth mat of 31 m diameter was fitted. The polarization error was measured with the aid of a loop transmitter at 6·3° elevation, the loop of which could be rotated about a horizontal axis in the plane of propagation so that its plane was 6° from the horizontal, thus giving at the direction-finder a field in which there was a small vertically-polarized and a large horizontally-polarized component. (Details of the techniques of measurement are given by Ross[1].) On one occasion when measurements were taken the total polarization error was found to be less than 3° at 13 Mc/s but over 30° at 20 Mc/s. On removal of the triatics and straightening of the main aerials the error at 20 Mc/s fell to 10°. This residual error is about three times as great as the values commonly found at lower frequencies; but it appears to be peculiar to the installation tested and is certainly unconnected with the aerial bending.

The inward bending of the aerials due to the sense aerial triatics was subsequently measured with the aid of a theodolite. When this was done it was found that the top of the west aerial was pulled in by about 15 cm and that of the east aerial by 3 cm. The polarization errors quoted were obtained with the loop transmitter to the south of the direction finder, so the slopes of the north and south aerials were immaterial. It was confirmed that the increase in error still occurred when the bending was done by ropes in the absence of the triatics and sense aerial; this ruled out the possibility of the error being due to the presence of the conducting material in the sense aerial and its supports.

MS accepted by the Editor, March 1955

3. Theoretical Evaluation of the Polarization Error

The figures just cited have been taken as giving the order of magnitude of the bending usually found, and have been used in theoretical calculations of the errors likely to be produced. Expressions for the polarization error are derived in Appendix 1. Now in these calculations a number of assumptions have been made to simplify the analysis. These will be stated here and later their validity and the effects of the modification of some of them will be discussed. They are as follows:

(1) All aerials are vertical except for the top portion (one quarter of the total length), which has a constant slope inwards towards the centre of the system.

(2) The slope of every portion of the aerials from the vertical is small, say less than 6°.

(3) The ground is perfectly reflecting, the phase change on reflection being 0° for vertically-polarized waves and 180° for horizontally-polarized waves.

(4) The transmitting current distribution for any of the Adcock aerials is of the form shown in equation (9) of Appendix 1. This quantity is required for the calculation of the voltage induced in an Adcock aerial which is receiving a signal.

(5) The transmitter is at a sufficient distance

from the Adcock for the incident radiation to be considered as a plane wave and for the surface wave to be neglected.

Of these assumptions (1) is certainly not exactly true, as the bottom part of the aerial cannot remain truly vertical if the aerial is pulled inwards at the top. However, by actual observation it was discovered that the aerials were bent very little except in the top section. No doubt this happens because each aerial consists of four separate sections, of equal length but each of a different radius, which are fitted together so that the smallest is at the top of the aerial. The diameters are 4, 3, 2 and $1\frac{1}{2}$ inches.

Assumption (2) is justified for the angles used in the calculation here, since the angle between the upper part of the aerial and the vertical is taken to be 4°.

The assumption (3) is probably nearly correct as regards the reflection coefficient for horizontally-polarized waves. However, the situation is more uncertain as regards the vertically-polarized waves, which will be reflected strongly by the earth mat but not so well by the ground. At 20 Mc/s, the frequency of the greatest interest here, the reflection coefficient for vertical polarization is only 0·4 and the phase change at reflection is a lag of about 160°; these figures apply to ground of conductivity 0·02 mho/m, which was the value obtained by measurement on the site. It is unlikely that the mat is responsible for giving the whole of the reflected field affecting the aerials, so the value of E_y given in equation (4) is probably an overestimate at all heights below the top of the aerial. The effect of introducing a reflection coefficient different from unity will be discussed later.

The transmitting current distribution assumed is probably reasonably near the truth; a more accurate formula would almost certainly complicate the analysis to an extent which would not be justified in view of the uncertainties in other parts of the computation. No change in this distribution will therefore be considered.

Assumption (5) is incorrect in both its parts if waves from the local loop transmitter are concerned. Of course, in the consideration of errors on a signal from a distant transmitter it is fully justified.

The magnitude of the polarization error calculated on this basis is shown plotted against frequency in Fig. 1. Here the parameter γ, the angle between the vertical and the line joining aerial tip to base, has been chosen to be 1°. The error certainly rises markedly with frequency, in a way very similar to that exhibited by the error in practice. However, it reaches a value at 20 Mc/s which is less than a third of the observed figure.

The effect of errors in the initial assumptions will now be considered. First of all, a change in the assumed shape of the Adcock aerial is not likely to affect the results seriously. This has been checked by carrying out the calculations for an aerial in the shape of a uniform rod clamped at one end and with a force acting at the other, and for an aerial with a constant slope of 1° to the vertical over its whole length. Both gave similar results to the original. The cantilever shape gave somewhat higher errors at the low frequencies, and the other a lower result at 20 Mc/s.

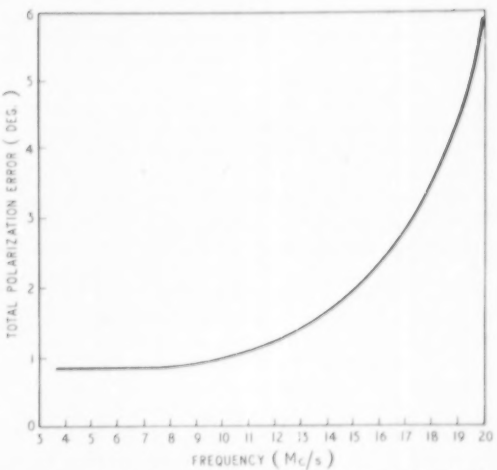

Fig. 1. *Polarization error for Adcock system with aerials bent inwards in the top portion, calculated from Equ.* (10) *with the overall slope* $\gamma = 1°$.

Because of its small size, the earth mat probably does not greatly affect the field at the aerials, particularly at their upper portions and with a low angle of elevation for the incident radiation. The results are therefore likely to be improved by assuming that the Adcock is situated on ground of finite conductivity. Now the introduction of a more realistic value of reflection coefficient for vertically-polarized waves has probably the biggest effect of all on the theoretical results. It we ignore the effect of the earth mat, this is given at 20 Mc/s by

$$R_V = 0{\cdot}4\, e^{-j160°}$$

Then the total field near the ground will be proportional to $(1 + R_V)$, approximately 0·6 in this case. The expression for the vertical field at any height y is

$$E_y = E_0 \sin \alpha \cos \delta \, [e^{j\beta y \sin \delta} + R_V\, e^{-j\beta y \sin \delta}]$$

where α is the tilt of the transmitter loop from the horizontal, δ is the angle of elevation of the transmitter, and β is the propagation constant of the waves in free space.

As $\beta y \sin \delta < 25^\circ$ always, it turns out that quite a good approximation to E_y is

$$E_y = 0{\cdot}6 E_0 \sin \alpha \cos \delta.$$

The cosine factor in the denominator of equation (10) makes little difference, so this means that the e.m.f. induced in the north–south aerial pair will be reduced by a factor $\frac{2}{0{\cdot}6} = 3{\cdot}3$ approx. (The transmitter is assumed to be at 180° azimuth.) This would increase the polarization error at 20 Mc/s by this factor, making it 19° approximately. Such a value is of the same order as the observed error, though it should be borne in mind that on the occasion when the slope of the aerial was measured it was only the west aerial which had a 1° slope, whereas the east aerial was nearly vertical. It is interesting to note that R_V will tend to $+1$ as the frequency is decreased, so that the decrease of error with decrease of frequency will be more rapid than is indicated by Fig. 1.

The difference between the plane wave considered and the actual direct and reflected waves arriving at the receiver is not likely to disturb the result seriously. The use of the loop transmitter, which is a reasonable approximation to a magnetic dipole, eliminates the 'proximity' errors (see Ross[1]) in the case of a perfect Adcock, since a magnetic dipole with its axis vertical radiates everywhere a purely horizontally-polarized field; the presence of a small amount of bending in the Adcock aerials does not affect this conclusion. The presence of the surface wave will, however, upset the results to a greater extent. Calculations indicate that the field E_y is altered by a factor of about 1·5 by the addition of the surface wave term. The surface wave is negligible in this case for horizontal polarization, so on this basis the error will be decreased from the value of 19° mentioned above to about 13°. The 19° error, of course, is the more appropriate to use in the case of a distant transmitter.

4. Effect on Practical Direction Finding

The bearing errors likely to be experienced in practice due to this aerial bending effect will not be so severe as might at first appear likely, since at 20 Mc/s radio waves will in general be incident on the direction finder at a low angle of elevation. This is because the critical frequency of the F_2 layer rarely approaches 20 Mc/s. Hence a wave incident on the direction finder of frequency near 20 Mc/s will be assumed to have an angle of elevation less than 20°; in such a case the horizontally- and vertically-polarized components are likely to be present in roughly equal amounts. Polarization errors have therefore been calculated for angles of elevation of 6·3°, 10° and 20°. They have been corrected for a finite ground conductivity (0·02 mho/m), and reduced by a factor of $\sqrt{2}$ to allow for averaging over all phase differences between horizontally- and vertically-polarized waves. The results appear in Table 1.

TABLE 1

Estimated Root-Mean-Square Polarization Error due to 1° Overall Aerial Bending for Downcoming Waves at Different Angles of Elevation

Angle of Elevation (deg.)	Frequency (Mc/s)	Root-Mean-Square Polarization Error (deg.)
6·3	16	0·6
10	16	0·5
20	16	0·8
6·3	20	1·8
10	20	1·3
20	20	3·0

An overall bend of 1° in an aerial implies that its tip is displaced inwards by 15 cm approximately. It should be noted that the errors at 20 Mc/s would be reduced if shorter aerials were used, and increased if the site were on ground of poorer conductivity.

5. Conclusions

It has been shown theoretically that if the aerials of an Adcock direction finder are bent inwards by the pull of triatics to give an overall slope of 1°, then polarization errors will be experienced which increase rapidly with frequency over 12 Mc/s. For a distant loop transmitter at 6·3° elevation and a loop tilt of 6° to the horizontal the total polarization error is 6° if the ground is perfectly reflecting, and 19° if the ground has conductivity 0·02 mho/m, even with this slight amount of aerial bending. The root-mean-square errors in practical direction finding with such an installation appear unlikely to exceed 3° at 20 Mc/s.

Acknowledgments

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The author is indebted to Marconi's Wireless Telegraph Co. Ltd., for the provision of facilities to perform the experiments described in this paper. He is also indebted to Dr. E. N. Bramley for details of some early experimental work on this effect. Assistance was rendered in the observational work and in the computations by Mr. J. Bell and Mr. J. Meadows.

APPENDIX 1

Polarization Error due to Bending of the Aerials

The principal symbols used are as follows.

x, y, z. Co-ordinate axes. The xy plane is a vertical

plane containing the transmitter and centre of the receiver, the x axis being horizontal and pointing in the direction from transmitter to receiver, the y axis being directed vertically upwards. The z axis is then horizontal (see Fig. 2). In the experimental arrangement considered here the transmitter is to the south of the receiver. The x axis therefore points north, and the z axis east.

E_x, E_y, E_z. Components of the electric field.

l. Total length of each Adcock aerial.

γ. Angle to vertical of a line joining the tip of an aerial to its base. Each aerial is assumed to be bent inwards towards the central axis of the system by the same amount.

α. The angle through which the axis of the transmitting dipole has to be rotated from the y axis towards the z axis in the yz plane to reach its actual position; the transmitter is considered here to be a magnetic dipole. As a loop transmitter is used to simulate the magnetic dipole, the plane of the loop will then be at an angle α from the horizontal. If $\alpha = 90°$ the field in the xy plane is vertically polarized; if $\alpha = 0°$ it is horizontally polarized.

δ. The angle of elevation of the transmitter from the direction finder. In the calculations the transmitter is assumed to be at a great distance from the direction finder, but in practice δ is measured from the centre of the system.

V_E, V_S, V_W, V_N. The open-circuit e.m.fs at the base of each aerial.

s. Aerial spacing in each Adcock pair.

λ. Wavelength of radio waves in free space.

$\beta = 2\pi/\lambda$.

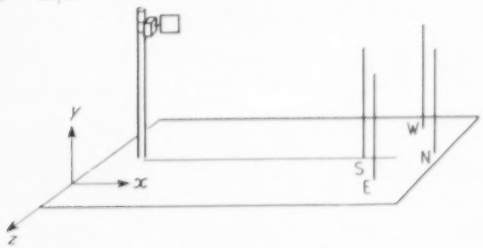

Fig. 2. *The co-ordinate system used.*

In accordance with the assumptions discussed in Section 3, the slope of the west aerial is taken to be

$$\left.\begin{aligned} \frac{dz}{dy} &= 0,\ 0 < y < \frac{3}{4}l \\ \frac{dz}{dy} &= 4\gamma,\ \frac{3}{4}l < y < l \\ \frac{dx}{dy} &= 0 \text{ everywhere} \end{aligned}\right\} \quad .. \quad .. \quad (1)$$

Now at height y the field along the aerial is $E_W(y)$, where

$$\left.\begin{aligned} E_W(y) &= E_y,\ 0 < y < \frac{3}{4}l \\ &= E_y \cos 4\gamma + E_z \sin 4\gamma,\ \frac{3}{4}l < y < l \end{aligned}\right\} \quad .. \quad (2)$$

Similarly for the east aerial which is assumed to be deflected inwards precisely as in the case of the west aerial.

$$\left.\begin{aligned} E_E(y) &= E_y,\ 0 < y < \frac{3}{4}l \\ &= E_y \cos 4\gamma - E_z \sin 4\gamma,\ \frac{3}{4}l < y < l \end{aligned}\right\} \quad (3)$$

Now suppose a plane wave is incident on the system from a distant magnetic dipole transmitter, and assume the ground is perfectly reflecting; the field components at a height y above the ground in the east-west plane can then be shown to be

$$\left.\begin{aligned} E_x &= 2jE_0 \sin\alpha \sin\delta \sin(\beta y \sin\delta) \\ E_y &= 2E_0 \sin\alpha \cos\delta \cos(\beta y \sin\delta) \\ E_z &= -2jE_0 \cos\alpha \cos\delta \sin(\beta y \sin\delta) \end{aligned}\right\} \quad (4)$$

Here E_0 is the amplitude of the field radiated by the transmitter in free space at a point on the meridian plane of its dipole and at the same distance from it as the Adcock.

The open-circuit e.m.f. in the west aerial is given by

$$V_W = \frac{1}{i_0}\int_0^l E_W(y)\, i_t(y)\, dy,$$

where $i_t(y)$ is the transmitting current distribution on the aerial when the current at the base is i_0. This method of calculating the voltage at the base terminal of a receiving aerial, which is based on considerations of reciprocity, is an exact one for a linear aerial in a general incident field; it has been given by Burgess[2]. A similar expression holds for the east aerial. In this problem the e.m.f. produced in the east-west Adcock pair is required. This is

$$\begin{aligned} V_W - V_E &= \frac{1}{i_0}\int_0^l [E_W(y) - E_E(y)]\, i_t(y)\, dy \\ &= \frac{2\sin 4\gamma}{i_0}\int_{3l/4}^l E_z(y)\, i_t(y)\, dy \quad .. \quad (5) \end{aligned}$$

If the north-south aerials have the same shape

$$\begin{aligned} E_S(y) &= E_y \exp\left(\frac{j\beta s\cos\delta}{2}\right),\ 0 < y < \frac{3}{4}l \\ &= (E_y\cos 4\gamma + E_z \sin 4\gamma)\exp\left(\frac{j\beta s\cos\delta}{2}\right),\ \frac{3}{4}l < y < l \\ & \quad .. \quad .. \quad (6) \end{aligned}$$

$$\begin{aligned} E_N(y) &= E_y \exp\left(-\frac{j\beta s\cos\delta}{2}\right),\ 0 < y < \frac{3}{4}l \\ &= (E_y\cos 4\gamma - E_z \sin 4\gamma)\exp\left(-\frac{j\beta s\cos\delta}{2}\right),\ \frac{3}{4}l < y < l \\ & \quad .. \quad .. \quad .. \quad (7) \end{aligned}$$

$$\begin{aligned} \therefore\ V_S - V_N &= \frac{1}{i_0}\int_0^l [E_S(y) - E_N(y)]\, i_t(y)\, dy \\ &= \frac{1}{i_0}\cdot 2j\sin\left(\frac{\beta s\cos\delta}{2}\right)\int_0^l E_y\, i_t(y)\, dy \\ &\quad + \frac{1}{i_0}\cdot 2\sin 4\gamma\cos\left(\frac{\beta s\cos\delta}{2}\right)\int_{3l/4}^l E_z\, i_t(y)\, dy \\ & \quad .. \quad .. \quad .. \quad (8) \end{aligned}$$

The factor of $\cos 4\gamma$ has been neglected as it is so nearly equal to unity. E_z and E_y are given by equation (4) and from these it can be seen that E_z is smaller than E_y by a factor of $\tan\delta \tan(\beta y \sin\delta)$. The second term in equation (8) is reduced still more compared with the first by the factor $\sin 4\gamma$. The factor $\cot\left(\frac{\beta s\cos\delta}{2}\right)$ tends to increase the ratio of the second term to the first, but does not outweigh the $\tan\delta$ factor in the range of frequencies considered. Hence as γ is a small angle the second term in equation (8) may be neglected compared with the first. We may, therefore, write

$$V_S - V_N = \frac{2j\sin\left(\frac{\beta s\cos\delta}{2}\right)}{i_0}\int_0^l E_y(y)\, i_t(y)\, dy$$

To evaluate this integral and that in equation (5) it is necessary to assume a particular transmitting current distribution $i_t(y)$. This has been selected to be

$$i_t(y) = \frac{i_0 \sin\{\mu\beta\,(l-y)\}}{\sin(\mu\beta l)} \quad .. \quad .. \quad (9)$$

This is the common sinusoidal current distribution, but with the propagation constant β modified empirically by the factor μ to take into account the finite thickness of the aerial. As the actual aerial varies in radius with height the value of μ was selected by examination of the resonance points in curves of aerial impedance which were obtained on the equipment tested. The value giving the best fit with the observed resonances was $\mu = 1{\cdot}14$.

$$= \frac{1}{2\beta}\left[\sin(\beta l \sin\delta)\left\{\frac{1}{\mu + \sin\delta} + \frac{1}{\mu - \sin\delta}\right\} - \frac{\sin\left\{\frac{\beta}{4}(3l\sin\delta - \mu l)\right\}}{\mu + \sin\delta} - \frac{\sin\left\{\frac{\beta}{4}(3l\sin\delta + \mu l)\right\}}{\mu - \sin\delta}\right] \quad .. \quad (11)$$

The total polarization error ϵ of the system in these conditions is given by

$$\tan\epsilon = \frac{V_W - V_E}{V_S - V_N},$$

since both voltages are in phase, and there will be no quadrature error.

$$= \frac{\sin 4\gamma}{j\sin\left(\frac{\beta s\cos\delta}{2}\right)} \cdot \frac{\int_{3l/4}^{l} E_x(y)\, i_t(y)\, dy}{\int_0^l E_y(y)\, i_t(y)\, dy}$$

Substituting from equations (4) and (9)

$$\tan\epsilon = \frac{-\sin 4\gamma \cot\alpha \int_{3l/4}^{l} \sin(\beta y\sin\delta)\sin\{\mu\beta\,(l-y)\}\,dy}{\sin\left(\frac{\beta s\cos\delta}{2}\right)\int_0^l \cos(\beta y\sin\delta)\sin\{\mu\beta\,(l-y)\}\,dy} \quad .. \quad .. \quad (10)$$

The integrals have been evaluated with the following results.

$$\int_{3l/4}^{l} \sin(\beta y\sin\delta)\sin\{\mu\beta\,(l-y)\}\,dy$$

$$\int_0^l \cos(\beta y\sin\delta)\sin\{\mu\beta\,(l-y)\}\,dy$$

$$= \frac{1}{2\beta}\left\{\cos(\beta l\sin\delta) - \cos(\mu\beta l)\right\}\left(\frac{1}{\mu - \sin\delta} + \frac{1}{\mu + \sin\delta}\right) \quad .. \quad .. \quad .. \quad (12)$$

The formulae (10), (11) and (12) enable the error to be computed.

REFERENCES

[1] W. Ross, "The Specification and Measurement of Polarization Error in Adcock-Type Direction Finders", *Proc. Instn elect. Engrs*, Part III, 1949, Vol. 96, p. 269.

[2] R. E. Burgess, "Aerial Characteristics", *Wireless Engr*, 1944, Vol. 21, p. 154.

CIRCUITS WITH NON-LINEAR RESISTANCE

Calculation of Behaviour

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(University of Southampton)

IT may be of interest to examine some simple circuits containing diodes as non-linear resistances, and to compare their behaviour with that of the corresponding linear circuits.

The particular circuits to be considered consist of a diode in series with a supply of a constant e.m.f. and an impedance Z, which will be taken to be either a purely ohmic resistance, an inductance, or a capacitance, or finally a series combination of all three.

The diode as a non-linear resistance obeys Child's Law

$$i = kv^{3/2} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (1a)$$

or

$$v = (i/k)^{2/3} \quad (1b)$$

Let a constant e.m.f. E be switched on at $t = 0$.

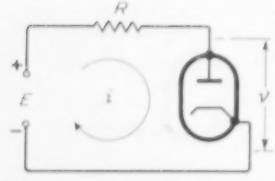

Fig. 1. *Linear and non-linear resistances in series.*

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Then we have

Case 1

The impedance is a purely ohmic resistance R (Fig. 1), and

$$Ri + v = E \quad .. \quad .. \quad .. \quad .. \quad (2)$$

Substituting for v from (1b) we get $i^{2/3} = k^{2/3}(E - Ri)$, or finally

$$-i^2 = k^2R^3(i - E/R)^3 \quad .. \quad .. \quad .. \quad (3)$$

To find i we can use the well-known graphical procedure shown in Fig. 2 in which the diode characteristic, given by (1a), is plotted; and also the straight line $i = (-1/R)(v - E)$ obtained from (2) with $1/R = \tan\alpha$. The ordinate of the intersection of the straight line and the characteristic is the solution of (3).

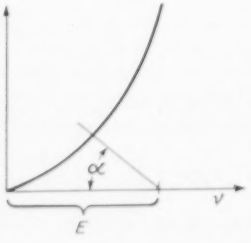


Fig. 2. *Graphical construction for Fig. 1.*

Case 2

The diode is in series with a pure inductance L (Fig. 3). The equation of the circuit is then $L\,di/dt + v = E$, which, by (1b), gives the differential equation for i

$$di/dt = E/L - (1/Lk^{2/3})i^{2/3} = K(E/KL - i^{2/3}),$$

where $K = 1/Lk^{2/3}$ .. .. (4)

Putting $E/KL = a^2$ .. .. .. (5)

we have, separating variables, $di/(a^2 - i^{2/3}) = K\,dt$, and obtain

$$Kt = \text{const} + \int di/(a^2 - i^{2/3}) \quad .. \quad .. \quad (6)$$

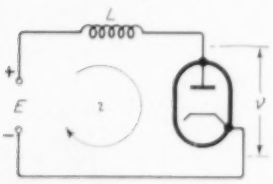


Fig. 3. *Inductance and non-linear resistance.*

If we put $i = x^3$, so that $di = 3x^2\,dx$, the integral on the right-hand side of (6) becomes

$$\int 3x^2\,dx/(a^2 - x^2) = 3\int dx\left\{a^2/(a^2 - x^2) - 1\right\}$$

$$= \frac{3a}{2}\log\frac{a + x}{a - x} - 3x$$

Thus (6) becomes

$$Kt = \text{const} + \frac{3a}{2}\log\frac{a + i^{1/3}}{a - i^{1/3}} - 3i^{1/3}.$$

Since, when $t = 0$, $i = 0$, the constant of integration is zero and

$$t = \frac{3a}{2K}\log\frac{a + i^{1/3}}{a - i^{1/3}} - \frac{3}{K}i^{1/3} \quad .. \quad .. \quad (7)$$

Equation (7) clearly satisfies the initial condition, and we see that $t \to \infty$ as $i^{1/3} \to a$, or as $i \to a^3$. By (5) and (4) we have $a^3 = kE^{3/2}$, so that $i \to kE^{3/2}$.

The limiting value of the current is the value which the current would have if E were applied directly to the diode; i.e., if the inductance L were absent. The same is true of the limiting value of the current in the case of a linear (ohmic) resistance in the place of the diode (although, of course, in that case the limit is determined by Ohm's not Child's Law). But the manner in which the limit is approached in the linear case is somewhat simpler according to the exponential function

$$(E/R)(1 - e^{-Rt/L}) \quad .. \quad .. \quad .. \quad (7')$$

Curve (a) in Fig. 4 shows the growth of the current i with time as computed from equation (7), while curve (b) shows the current in the linear case increasing according to the exponential function (7'). The curves show that in the non-linear case the current grows more slowly than in the comparable linear case. As can be seen by differentiating (7) and (7') the two curves (a) and (b) in Fig. 4 have a common tangent at the origin.

It may be added that in the linear case the ohmic resistance which was substituted for the diode was of such magnitude as to make the limiting values of the currents equal, i.e.,

$$E/R = kE^{3/2}, \text{ or } R = 1/kE^{1/2}.$$

Case 2a

If, when a current i_0 flows in the circuit of Case 2, the e.m.f. is short-circuited, we have $L\,di/dt + v = 0$, or by (1b): $di/dt = -i^{2/3}/Lk^{2/3}$. Separating variables, we get $di/i^{2/3} = -dt/Lk^{2/3}$, and, integrating: $i^{1/3} = \text{const} - t/3Lk^{2/3}$, where $t = 0$ at the moment of short circuit. Using the initial condition in this case, we find that the constant of integration is $i_0^{1/3}$ so that we get, finally: $i = (i_0^{1/3} - t/3Lk^{2/3})^3$ .. .. (7a)

The current is reduced to zero in the finite time $t = t_1 = 3Lk^{2/3}\,i_0^{1/3}$. This result is entirely different from that obtained with a linear resistance.

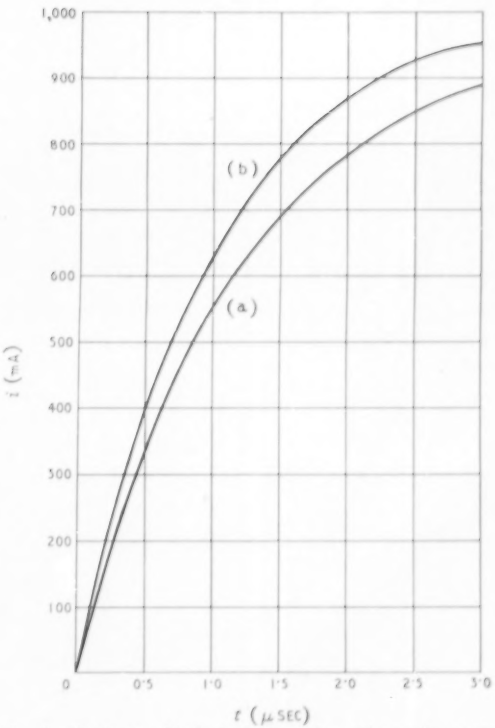


Fig. 4. *Response of inductance in series with non-linear (a) and linear (b) resistances.*

Case 3

The diode is in series with a capacitance C (Fig. 5). In this case we have

$$v + q/C = E \qquad (8)$$

where q is the charge on the capacitor.

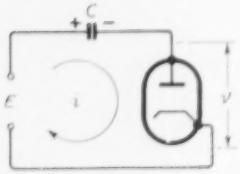


Fig. 5. *Capacitance and non-linear resistance.*

Eliminating v by means of (1b) we obtain the differential equation for q

$$i = dq/dt = k(E - q/C)^{3/2} = kC^{-3/2}(CE - q)^{3/2}.$$

Separating variables we have

$$dq(CE - q)^{-3/2} = kC^{-3/2}\,dt,$$

and hence

$$(CE - q)^{-1/2} = \tfrac{1}{2}kC^{-3/2}\,t + \text{const.}$$

Since, when $t = 0$, $q = 0$, the constant of integration is $(CE)^{-1/2}$, so that, putting

$$2C^{3/2}/k = \alpha \qquad (9)$$

we have $(CE - q)^{-1/2} = (1/\alpha)(t + \alpha/\sqrt{CE})$ and, solving for q,

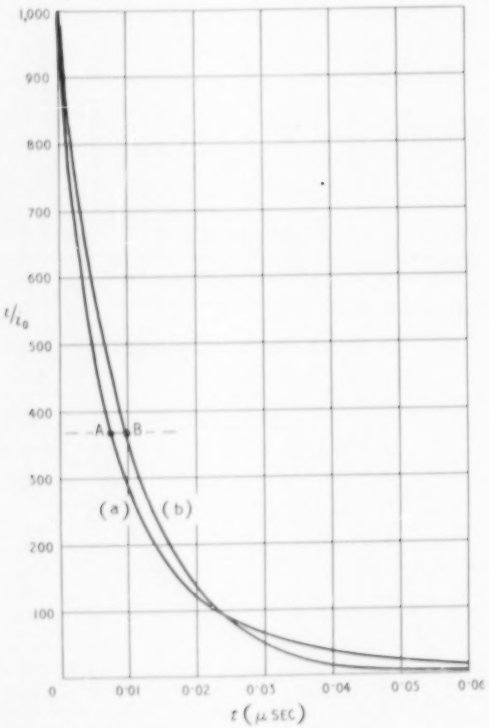


Fig. 6. *Charging current of capacitance through non-linear (a) and linear (b) resistance.*

$$q = CE - \alpha^2/(t + \alpha/\sqrt{CE})^2 \qquad (10)$$

By differentiation we obtain the current

$$i = 2\alpha^2/(t + \alpha/\sqrt{CE})^3 \qquad (11)$$

When $t = 0$, (10) gives $q = 0$, as required by the initial condition. But the initial value of the current is, by (11) and (9), $kE^{3/2}$, in agreement with (1a), since, when $q = 0$, we have, by (8), $v = E$.

As $t \to \infty$, we have $q \to CE$ and $i \to 0$. The asymptotic limits of q and i are the same as those in the case of a linear resistance in series with a capacitance, but the manner in which the limits are approached is different: $q = CE(1 - e^{-t/CR}) \to CE$ as $t \to \infty$, and $i = (E/R)\,e^{-t/CR} \to 0$ as $t \to \infty$ (11′)

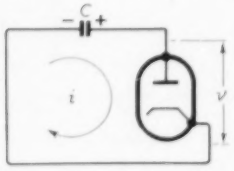


Fig. 7. *Discharge of capacitance through a non-linear resistance.*

Curve (a) in Fig. 6 shows the current according to equation (11), while curve (b) shows the current in the comparable linear case (11′). The abscissae of the points A and B are the times for the two cases when the current has been reduced to $1/e$ times the (common) initial value. It appears that the current in our non-linear case decreases more rapidly than in the linear case, at least up to a certain time (0·025 μsec in the numerical example represented in Fig. 6), after which the exponential current (11′) of the linear case is the smaller one of the two.

Case 3a

The difference in the manner in which the limiting values of charge and current are approached, with a linear and with our non-linear resistance in series with a capacitance, is shown more clearly when the diode is connected to a capacitor C bearing an initial charge q_0. With the polarity shown in Fig. 7 we must put

$$i = -dq/dt \qquad (12)$$

which gives, by (1a) and with $v = q/C$,

$$i = -dq/dt = k(q/C)^{3/2}.$$

Separating variables we have $-dq/q^{3/2} = k\,dt/C^{3/2}$, which, integrated, gives $1/q^{1/2} = \text{const} + kt/2C^{3/2}$. By the initial condition the constant of integration is $1/q_0^{1/2}$, and so

$1/\sqrt{q} = kt/2C^{3/2} + 1/\sqrt{q_0} = (1/\alpha)(t + \alpha/\sqrt{q_0})$, where α is given by (9). Then we have finally the charge on the capacitor

$$q = \alpha^2/(t + \alpha/\sqrt{q_0})^2 \qquad (13)$$

By equation (12) we obtain the current

$$i = 2\alpha^2/(t + \alpha/\sqrt{q_0})^3 \quad .. \quad .. \quad (14)$$

We see from (13) and (14) that both q and i tend to zero as $t \to \infty$, as charge and current also do when a linear resistance R takes the place of the diode. But whereas in the linear case both q and i tend to zero according to the same exponential function $e^{-t/CR}$, in our non-linear

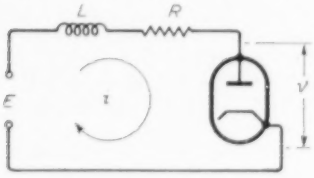


Fig. 8. *Inductance with both linear and non-linear resistance.*

case the charge and the current tend to zero as the square and the cube, respectively, of $1/(t + \alpha/\sqrt{q_0})$. In contrast to the linear case, charge and current are not in a constant ratio, which reflects the non-linearity of the diode characteristic; i.e., the fact that as q tends to zero (and with it the voltage q/C across the diode) the resistance of the diode increases.

Case 4

The e.m.f. E is switched on, at $t = 0$, in series with an inductive resistance L, R and the diode (Fig. 8). Kirchhoff's Law and (1b) give a differential equation for i, which can be separated and gives

$$t = L\int di/\{E - Ri - (i/k)^{2/3}\} \quad .. \quad (15)$$

Putting $i = x^3$, so that $di = 3x^2dx$, the integral in (15) takes the form $\int x^2\,dx/(a - bx^2 - x^3)$. By the choice of particular numerical values for R, L, k, E, it is possible to factorize the denominator and so express t as a sum of logarithms, one of which tends to infinity as x approaches a certain number. Thus $t \to \infty$ as x tends to a certain (numerical) limit.

Moreover, putting the chosen numerical values of the constants into equation (3), the numerical limit of the current of Case 4 satisfies (3). (For a constant current L has no effect and Cases 1 and 4 are identical.)

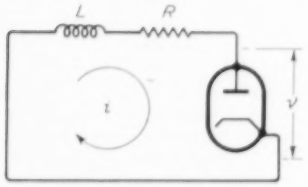


Fig. 9 (left). *Decay of current in inductive circuit.*

Case 4*a*

More interesting conditions exist when an initial current i_0 flows in a circuit consisting of the diode in series with an inductive resistance L, R (Fig. 9).

The equation of this circuit is

$L\,di/dt + Ri + v = 0$ which is, by (1b),

$di/dt = -Ri/L - i^{2/3}/k^{2/3}L =$

$-(R/L)(i + i^{2/3}/Rk^{2/3})$

Separating variables and putting

$$1/Rk^{2/3} = a$$
$$i = x^3 \quad .. \quad .. \quad (16)$$

we get $dx/(x + a) = -Rdt/3L$ and hence the integral $\log(x + a) = \text{const} - Rt/3L$. Since when $t = 0$, $i = i_0$ (and hence $x = x_0$) the constant of integration is $\log(x_0 + a)$ and so

$$(x + a) = (x_0 + a)e^{-Rt/3L} \quad .. \quad .. \quad (17)$$

As in Case 2a, we see in this case, too, that the current i becomes zero ($x = 0$) in the finite time

$$t = t_1 = \frac{3L}{R}\log\frac{x_0 + a}{a} \quad .. \quad .. \quad (18)$$

where $x_0^3 = i_0$ and a is as defined in (16).

Fig. 10 illustrates (17) and (18).

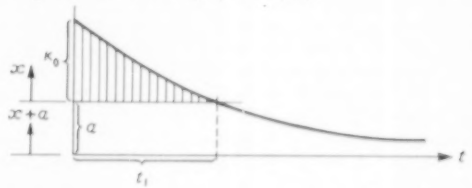


Fig. 10. *Graph of current decay for the circuit of Fig. 9.*

Case 5

The diode is connected in series with an inductance L, a resistance R, and a capacitance C. At $t = 0$ the constant e.m.f. E is switched on, the initial charge and current being zero (Fig. 11).

The equation of the circuit is

$$L\,di/dt + Ri + q/C + v = E \quad .. \quad (19)$$

Substituting for v by (1b), putting $R/L = 2\mu$; $1/Lk^{2/3} = K$; and $1/CL = \alpha^2$ and moving the constant term to the left-hand side, (19) becomes

$$d^2q/dt^2 + 2\mu\,dq/dt + K(dq/dt)^{2/3} + \alpha^2(q - CE) = 0 \quad .. \quad .. \quad .. \quad (20)$$

We now introduce new variables x, θ by putting

$$(q - CE) = (1/\alpha)(K/2\mu)^3x \quad .. \quad .. \quad .. \quad (21)$$

and $t = \theta/\alpha$ so that $d/dt = \alpha d/d\theta$. Denoting

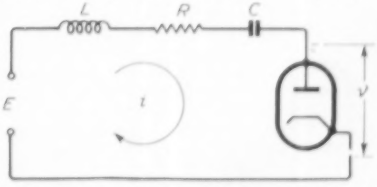


Fig. 11 (right). *Damped resonant circuit including a non-linear resistance.*

differentiation with respect to θ by primes we obtain (20) in the form

$$x'' + \epsilon(x' + x'^{2/3}) + x = 0 \quad .. \quad .. \quad (22)$$

where $\epsilon = 2\mu/\alpha$.

Equation (22) is of the type $x'' + \phi(x') + x = 0$, of which van der Pol's equation is the special case $\phi(x') = -x' + x'^3$. We are here considering the special case $\phi(x') = \epsilon(x' + x'^{2/3})$.

From (21) it is clear that, whereas x' is proportional to the current i, x itself is proportional, not to the charge q, but to $(q - CE)$, so that the initial conditions, $i = 0$ and $q = 0$ when $t = 0$, are now $x' = 0$ and

$$x = -CE\alpha(2\mu/K)^3 \text{ when } \theta = 0 \quad .. \quad (23)$$

If the coefficient ϵ is sufficiently small, the equation is that of a damped oscillation with the damping term $\phi(x') = \epsilon(x' + x'^{2/3})$.

If $\phi(x')$ were linear in x', the solution would be the familiar one of an exponentially decreasing sine or cosine.

But owing to the term $x'^{2/3}$ in $\phi(x')$, equation (22) is non-linear, and its solution in general terms becomes impracticable. We seek, therefore, a numerical solution, assuming particular values of L, R, C, and the constant k of equation (1a).

If we take, as special values, $R = 10$ ohms, $L = 10^{-4}$ henry, $C = 10^{-10}$ farad, and $k = 10^{-3}$, we have: $1/CL = \alpha^2 = 10^{14}$ (hence $\alpha = 10^7$); $R/L = 2\mu = 10^5$; $1/Lk^{2/3} = K = 10^6$; and $2\mu/\alpha = \epsilon = 10^{-2}$. Thus the equation (22) becomes

$$x'' + 10^{-2}(x' + x'^{2/3}) + x = 0 \quad .. \quad .. \quad (24)$$

In any one of the standard methods of numerical integration it is necessary first to choose an interval h of the independent variable (i.e., in our case, of θ), then to calculate, with the help of Taylor's series, a few values of x and x', beginning with the initial values, and from the differential equation the corresponding values of x''. Thereafter the formulae given in the particular method chosen will yield further values of x and x', and again the differential equation gives the corresponding value of x'', and so on.

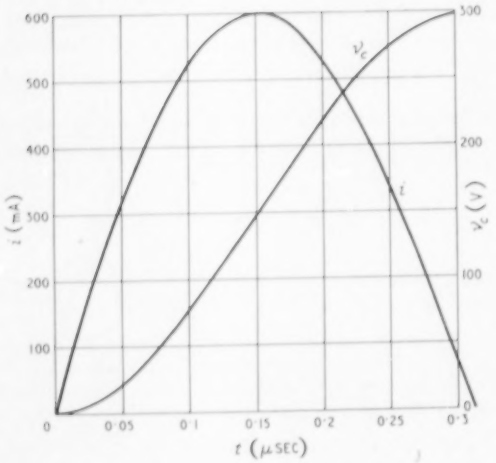


Fig. 12. *Current and voltage for the circuit of Fig.* 11.

With our equation (24), however, a difficulty arises in the calculation of the first few values, owing to the fact that the non-linear power of x' is fractional. For in Taylor's series for x and x':

$$x(\theta + h) = x(\theta) + hx'(\theta) + \tfrac{1}{2}h^2 x''(\theta) + \ldots$$
$$x'(\theta + h) = x'(\theta) + hx''(\theta) + \tfrac{1}{2}h^2 x'''(\theta) + \ldots$$

we have, by differentiating (22): $x''' = -x' - x''(1 + \frac{2}{3}x'^{-1/3})$ which, initially, is not finite, since there occurs a negative power of x', and $x' = 0$ when $\theta = 0$.

This difficulty can be overcome if, instead of beginning the computation at $\theta = 0$, when $x' = 0$, we start at the maximum value of x' (i.e., at the current maximum) when $\theta = \theta^*$, and carry out the integration first 'inwards' (i.e., taking h negative) and then 'outwards' (i.e., with h positive).

In either direction the integration need only be continued until a zero of x' is reached, since the current through the diode cannot reverse its sign.

The value of x corresponding to the zero of x' reached by integrating 'inwards' is then the initial value x_0 of x.

But as the choice of the starting point of the computation (maximum current) is arbitrary, the freedom to choose the applied e.m.f. E [which determines x_0 by (21)] is lost. If, in any concrete practical problem, E is given, the only course open is to carry out the 'inward' computation several times, beginning with different maximum values of x' until we arrive at values x_0 corresponding to values of E which are near enough to the given e.m.f.

The current, and so x', is a maximum when $x'' = 0$. Then, by (24), $x = -10^{-2}(x' + x'^{2/3})$. So, for a maximum value of 300×10^{-6} for x' which is equivalent to a current of 300 mA, the corresponding value of x is found to be $-47{\cdot}81 \times 10^{-6}$. These 'starting' values for x and x' lead to the 'initial' value of x (when $x' = 0$) of between $-340{\cdot}45 \times 10^{-6}$ and $-339{\cdot}01 \times 10^{-6}$, or about -340×10^{-6} which, by (21), implies an e.m.f. $E = 340$ volts.

Instead of using one of the more elaborate formulae for computation (e.g., Milne-Simpson's, Karman's, etc.), which yield sufficiently accurate results with comparatively larger, and therefore fewer, intervals h, but require four values of x and x' to be calculated before the step-by-step application of the formulae, the simple formulae

$$x_n = x_{n-2} + 2hx'_{n-1} \text{ and } x'_n = x'_{n-2} + 2hx''_{n-1} \quad .. \quad .. \quad .. \quad (25)$$

have been used, which require only two values of x and x' to be calculated beforehand, and which

are quite adequate provided the interval h is chosen sufficiently small.

To choose h, let us omit, for a moment, the term $\phi(x')$, which reduces the differential equation (24) to the type $x'' + \omega^2 x = 0$, with $\omega = 2\pi/\tau = 1$, so that the period (in units of the θ-time scale) is $\tau = 2\pi \approx 6$. Taking $h = 10^{-1}$, we have about 60 intervals in one period, or, since we carry out the computation over one half-period only (between two zeros of the current), there are about 30 points to be calculated.

Taking $h = 5 \times 10^{-2}$ doubles the number of points to be calculated, but makes the practical computation very convenient (since then $2h = 10^{-1}$) and increases the accuracy.

The graph (Fig. 12) shows the current i, obtained from x' by (21), in mA, and $q/C = v_c$ the voltage across the capacitor, q having been obtained from x by (21). The time-scale has also been changed from θ to t, so that $h = 0{\cdot}01$ μsec.

If we compare this result with that of the linear case leading to the same maximum capacitor voltage, we find that the current maximum is a little greater and occurs slightly earlier. The capacitor voltage, too, is a little higher in the linear case, except at the ends of the time-range, where the voltages in the two cases coincide.

It may be added that, if our computation is carried on beyond the zero of the current, the curve obtained is similar to that of an oscillation with linear damping. It would represent the non-linear analogue to a damped oscillation, which could be realized by replacing our one diode by an anti-parallel combination of two.

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

X-Ray Production by Magnetron

SIR,—While investigating the power-handling capacity of waveguide rotating joints (W.G.16) it was discovered that results varied considerably depending on whether the rotating-joint assembly was connected to the magnetron (type 4J50) by means of a straight or curved length of waveguide. When a straight piece of waveguide was used, breakdown occurred at a level of 150 kW peak whereas with a curved length of connecting waveguide breakdown was in excess of 240 kW peak. The curve in the waveguide need only be sufficient to impede a direct line of sight between the magnetron window and the waveguide assembly under test. There was no difference in match for these two conditions.

By interposing an H-plane bend containing a long tube soldered into the narrow face of the bend it was possible to observe the magnetron window without upsetting the full power X-band conditions (the tube is being operated below cut off). A radiation monitor applied to the end of the tube showed the presence of X-rays. These are 'copper' X-rays having an energy potential of about 8·5 kV. The exposure of X-ray film at the end of the inspection tube confirmed the existence of X-rays.

It is concluded therefore that any equipment using a 4J50 type of magnetron and a waveguide system in W.G.16 waveguide should include a bend in the waveguide (effectively an X-ray filter) as close to the magnetron as possible.

The X-rays are also emitted through the cathode stem of the magnetron in a cone-shaped beam whose axis is that of the cathode stem. Roughly, the limits of the cone-shaped beam are the same as those which define the visibility of the magnetron heater between the cathode shield and the sole plate. The intensity of the X-rays from this source is less than obtained in the waveguide output and is very difficult to measure. Difficulty arises because of the high voltages applied to the magnetron cathode and because the radiation intensity is too high to be measured conveniently by a scintillation counter but too low to be measured by the type of ionization chamber readily available. The X-rays are so 'soft', however, that they will not penetrate to the bone marrow and any harmful effects will be confined to unprotected (e.g., by normal clothing) skin. Any such effects are cumulative (over a period, say, of a year) and it is recommended that steps are taken to prevent the exposure of operating personnel to this radiation.

It should be noted also that, in a typical equipment, the cathode-stem radiation may illuminate other components, such as pulse transformers and delay networks, thus facilitating corona or other breakdown in these components.

A. C. WESLEY

Radar Department, Ferranti Ltd.,
Edinburgh.
28th November 1955.

Transient Response Calculation

SIR,—I was interested to see Mr. D. G. Sarma's article "Transient Response Calculation" in *Wireless Engineer* (July 1955) which extended the method of linear segments for estimating the transient response associated with a given gain–frequency characteristic of a minimum-phase network. A comparison between this method and a method I suggested[1] using standard gain–frequency curves[2] might be of interest.

The main disadvantage of Mr. Sarma's method is that the number of poles and zeros in the approximating function is much greater than the number in the original function. Thus, in the given example a total of 13 poles and zeros are used to approximate a function having only 5. This means that a considerable amount of labour is necessary both in calculating the residues at the poles and in obtaining numerical values from a complicated time function. On the other hand, by using the standard gain–frequency curves a very reasonable approximation can often be obtained using a smaller number of poles and zeros than are present in the original function.

A method of approximation using straight-line segments or semi-infinite slopes, in which the straight lines can be drawn arbitrarily, will doubtless more quickly provide some sort of approximation to a given gain characteristic than a method using more complicated

approximating functions. However, the full saving in time on this particular point will not be realized in Mr. Sarma's method since the straight-line segments cannot be put in arbitrarily. Further, some numerical calculation is necessary to obtain the poles and zeros corresponding to the chosen linear segments.

The method using the standard gain–frequency curves has the advantage of being easily extended to the non-minimum-phase case, the procedure being as follows:—

(a) Use the standard gain–frequency curves, $10 \log_{10} [x^2 + 2x \cos 2\delta + 1]$, to obtain an approximation to the given gain–frequency characteristic.

(b) Use the standard phase–frequency curves[1], $\arctan[2x \cos \delta/(1 - x^2)]$, to obtain the corresponding minimum-phase characteristic, all poles and zeros being taken to lie in left-half plane. As the given system function is non-minimum-phase the phase–frequency characteristic so obtained will not be identical with the given phase–frequency characteristic.

(c) By transferring appropriate singularities (zeros in the case of a gain-type system function) to the right-half plane or by adding the phase characteristics corresponding to all-pass networks (*twice* the standard phase–frequency curves) pole-zero positions are finally obtained that correspond to good approximations to both the given gain–frequency and phase–frequency characteristics A *good* approximation is hence obtained for the response to a given transient. The total number of zeros to be transferred, or the total number of all-pass networks to add, is found by noting the difference between the value of the phase at frequency $+ \infty$ minus the phase at frequency $+ 0$ for the given phase characteristic and the minimum-phase characteristic.

O. P. D. Cutteridge

Electrical Engineering Dept.,
Faculty of Technology,
University of Manchester.
19th November 1955.

REFERENCES

[1] O. P. D. Cutteridge, "A Graphical Contribution to the Analysis and Synthesis of Electrical Networks", *Proc. Instn elect. Engrs*, Oct. 1953, Vol. 100, Part IV, pp. 83–90.

[2] J. M. Linke, "A Graphical Approach to the Synthesis of General Insertion Attenuation Functions", *Proc. Instn elect. Engrs*, 1950, Vol. 97, Part III, pp. 179–187.

MEETINGS

I.E.E.

9th January. "The Efficient Use of Technical Personnel", discussion to be opened by the President, J. Eccles, C.B.E., B.Sc.

11th January. "Pulse-Time Modulation Terminals for Music Transmission over Radio Links", by R. F. Rous, B.Sc.

16th January. "Advanced Courses for Engineers in Industry", by Willis Jackson, D.Sc., D.Phil., F.R.S., H. D. Morgan, M.Sc.(Eng.) and Professor G. F. Mucklow, D.Sc. Joint meeting with the Institutions of Civil and Mechanical Engineers at the Institution of Mechanical Engineers, 1 Birdcage Walk, Westminster, London, S.W.1.

23rd January. "Particle Accelerators", by E. L. Wiblin, M.A.

These meetings will commence at 5.30 and will be held at the Institution of Electrical Engineers, Savoy Place, Victoria Embankment, London, W.C.2, except where otherwise stated.

Brit.I.R.E.

5th January. "Domestic Tape Recording Applications with Special Reference to Stereophonic Reproduction", by M. B. Martin and D. L. A. Smith, B.Sc.(Eng.).

25th January. Symposium on Electronic Methods of Pictorial Reproduction:—"Facsimile Communication", by H. F. Woodman and P. H. J. Taylor; "Facsimile Transmission of Weather Charts", by J. A. B. Davidson; "Tone Reproduction with Electronic Stencils", by R. Lant, Ph.D. and "Electronic Engraving", by G. S. Allen.

These meetings will commence at 6.30 and will be held at the London School of Hygiene and Tropical Medicine, Keppel Street, Gower Street, London, W.C.1.

The Television Society

19th January. "Non-Entertainment Aspects of Television", The Fleming Memorial Lecture by Professor J. D. McGee, at 7 o'clock at the Royal Institution of Great Britain, 21 Albemarle Street, London, W.1. Admission by ticket only.

Radar Association

13th January. "Radio Astronomy", by Professor A. C. B. Lovell, at 7.30 at the Anatomy Theatre, University College, Gower Street, London, W.C.1.

STANDARD-FREQUENCY TRANSMISSIONS

(Communication from the National Physical Laboratory)

Values for November 1955

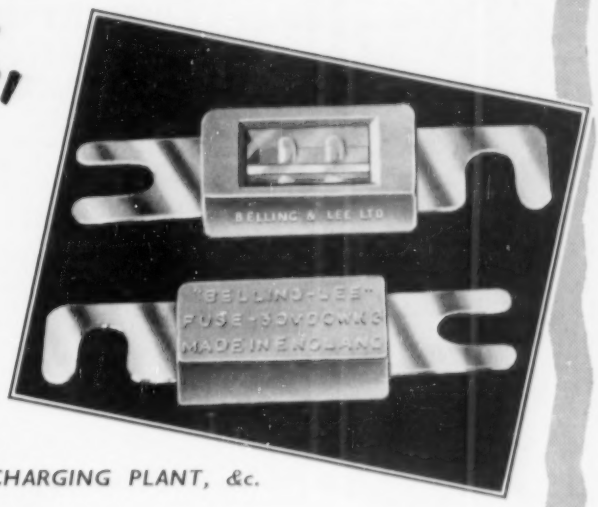
Date 1955 November	Frequency deviation from nominal: parts in 10^8	
	MSF 60 kc/s 1429-1530 G.M.T.	Droitwich 200 kc/s 1030 G.M.T.
1	N.T.	+5
2	N.T.	+1
3	N.T.	+1
4	N.T.	+1
5	N.T.	+1
6	N.T.	+1
7	+0·1	+1
8	+0·1	+1
9	+0·1	+1
10	+0·1	+1
11	+0·1	+1
12	+0·1	+1
13	+0·1	+1
14	+0·1	+1
15	0·0	+1
16	+0·1	+2
17	+0·1	+1
18	+0·1	+1
19	+0·2	+1
20	+0·2	+1
21	+0·1	+1
22	+0·2	+1
23	+0·1	+1
24	+0·1	+2
25	+0·1	+2
26	+0·1	+2
27	+0·1	+2
28	+0·1	+2
29	+0·1	+2
30	+0·1	+3

The values are based on astronomical data available on 1st December 1955.

N.T. = No Transmission.

The MSF 60-kc/s aerial system was undergoing maintenance during the first six days of the month.

Heavy Current, Low Voltage, Fuse-Links



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SPECIFICATION: The fuselink is to be capable of breaking currents of up to 3,000 amperes at 30V. d.c. (2,500 amperes at 30V. d.c. for the 35A and 50A ratings) without shattering. The arcing time shall not exceed 0.003 seconds at 3,000 amperes.

The fuse is intended as a direct replacement for the American "Current Limiter" used on 28V. d.c. aircraft power supplies, and for this purpose has received the approval of the Air Registration Board under their reference No. E.3217.

It is also suitable for the protection of battery operated vehicles, heavy current rectifier output circuits, low voltage furnaces and other similar d.c. applications.

RATINGS: 35A, 50A, 80A, 100A, 130A, 150A, 200A, 225A, and 275A. The range will be extended in four further steps to 500A maximum.

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L.693. 1¼" × ¼". Ratings: 0.25, 0.5, 1, 2, 3, 5 & 7A.
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It is expected that their d.c. category of duty will comply with B.S. 88, d.c.4. 230V.

L.754. ¾" × 3/16". Ratings: 0.025, 0.050, 0.100, 0.250, 0.500, 1 & 2A.
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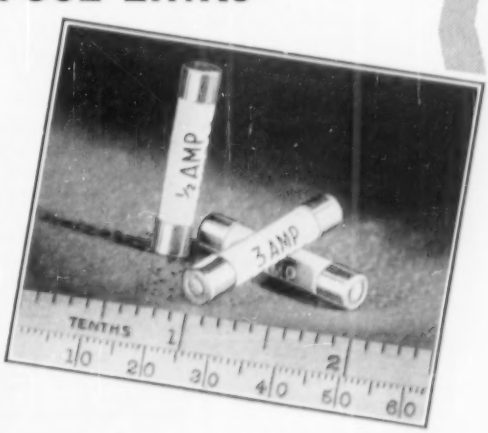
Also available but not to the above specifications:

L.760. 1¼" × ¼" Ceramic, unfilled, designed to blow within 10 secs. on 100% overload.



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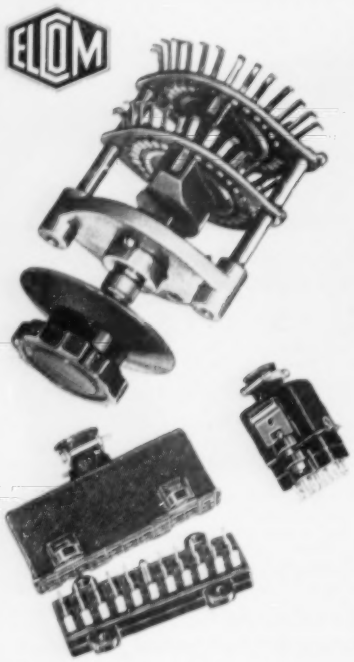

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On the lower D.C. ranges, this instrument approaches conditions of an electrostatic voltmeter and on A.C. ranges a measuring diode contained in an external probe, is provided. To minimize the effect of the metal body of the probe, the insulated terminal head can be replaced by a spike. All-range indicator consists of a 5-inch meter fitted with knife-edge pointer and mirror scale. Send for leaflet giving full specification.

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Volt -	0-300 *mV*	0-1 *V*
Ranges -	0-1 *V* - 0-300 *V*	0-3 *V* - 0-100 *V*
Accuracy	±2% *of f.s.d.*	±2% *of f.s.d.*
Input -	*For the first five ranges, 25 megohms*	*20 megohms up to medium R.F., on all ranges*

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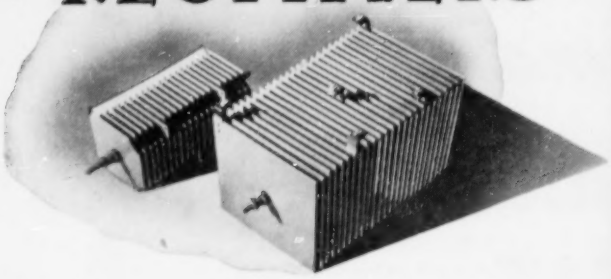

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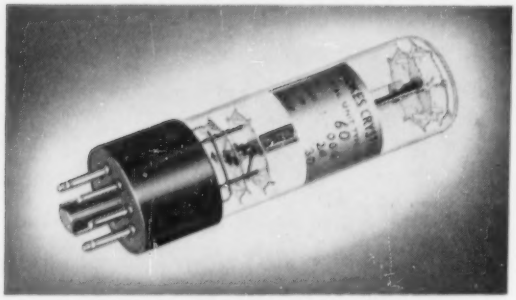

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Plessey 100-Watt Heavy Duty Vibrator

Developed in conjunction with S.R.D.E.

Specification

Operating Voltage	24v. nominal*
Operating Frequency	110 c/s nominal
Drive Coil Power	5 watts max.
Contact Closure	
Primaries	72%—84%
Secondaries	62½—74½%
Working Life	1,000 hrs. nominal at 20° C.
	500 hrs. nominal at 70° C.
Voltage Proof (at pressures down to 70 mm. Hg.)	750v. r.m.s. 50 c/s. between all parts not electrically connected.
Input Rating (24v. nominal supply)	5—8 amps.
Temperature Category	40/70 (— 40° C. to + 70° C.)
Humidity Class	H.1 meets requirements of R.C.S.172.
Bumping	Meets requirements of R.C.S.172.
Weight	12.6 oz.

* A 12-volt version of this vibrator is at present under test and will become available as soon as conditions of life and rating have been determined.

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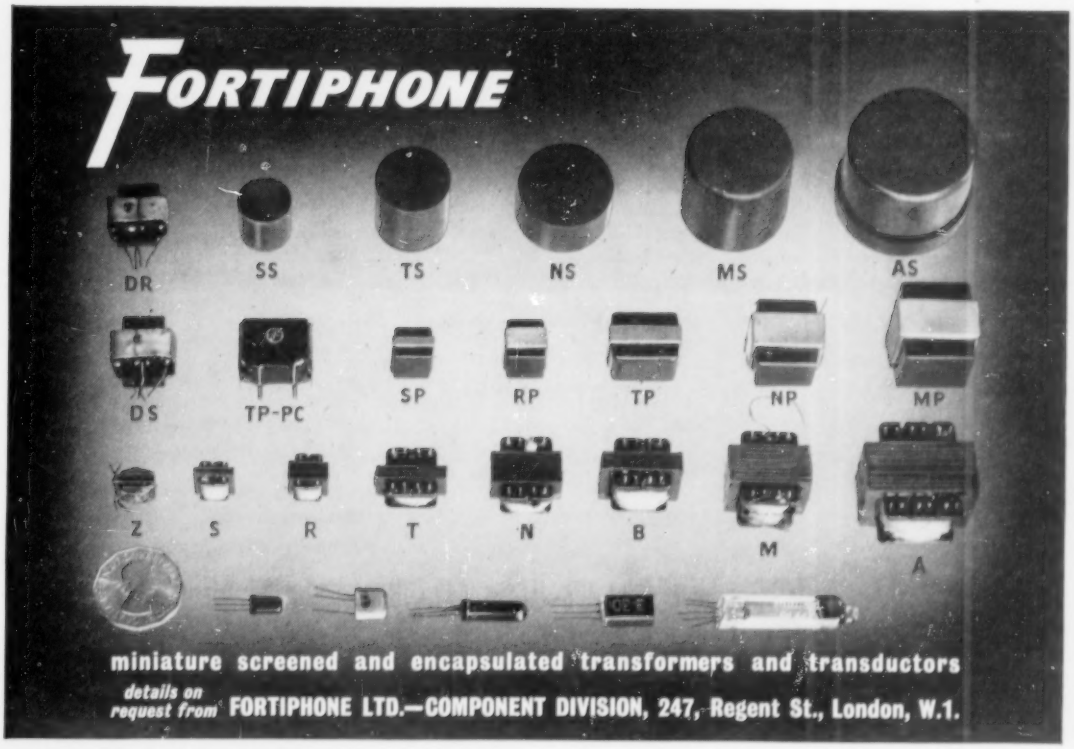

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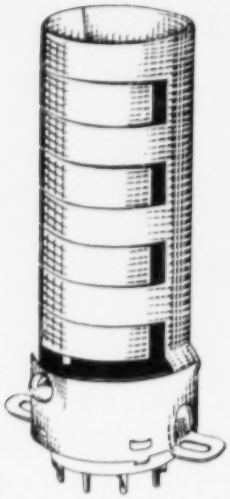

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I *Accurately calibrated time and voltage measurement.*
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V *Robust and accessible construction.*

See what you measure!

Two Channel Wide Band D.C. 10 Mc/s. Rise time 0·035 μsec in each channel with identical amplifiers. Versatile time base with calibrated sweep times from 0·2 second to 2·0 μsecs per cm.

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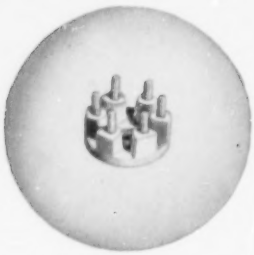

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Diameter ·5"—Height ·35"
Fixing hole diameter ·12"

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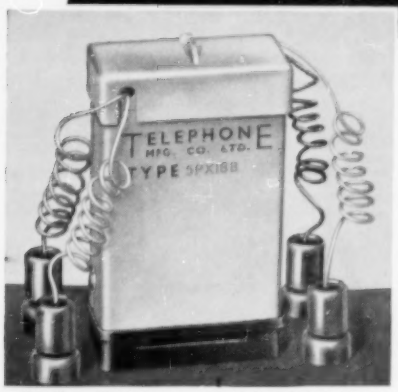

The Type 5PX Carpenter Polarised Relay is fitted with platinum contacts to reduce thermal noise, and has flying contact leads to reduce "pick-up" in contact circuit due to the coil. DIMENSIONS : *Height* 2.5 *in. Width* 1.6 *in. Depth* 0.8 *in. Approx. weight* 4.8 *oz.*

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The fast operating Carpenter Polarized Relay Type 5PX is being used very satisfactorily in a wide variety of control, recording and test circuits where the available initiating voltage is extremely low and must be greatly amplified before it can be used. The relay enables this to be done efficiently and economically by converting ("chopping") d.c. input signals to a square-wave alternating voltage which may then be amplified simply in an a.c. amplifier.

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Candidates should be A.M.I.E.E. or have passed or be exempt from Parts A and B of the Institution's examination. A sound knowledge of the theory of telecommunications engineering is necessary and extensive experience of its application.

Apply in writing to the Director of Recruitment, Colonial Office, Great Smith Street, London, S.W.1, giving briefly age, qualifications and experience. Mention the reference number BCD 108/15/03.

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have a number of vacancies in the Estimating Department at their Mitcham plant. The posts available are for:—

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Applicants should have a sound knowledge of theory and practice of electronic or light electro-mechanical engineering. Experience in either design, development, maintenance, inspection, production or operation of electronic, electro-mechanical, radio or radar installations and equipment or in design of jigs and tools would be an advantage. All applicants should have served a recognized engineering apprenticeship or have had equivalent training in an appropriate trade. For higher grade posts applicants should either be Corporate Members of one of the Institutions of Civil, Mechanical, or Electrical Engineers or have exempting qualifications. Those lacking higher academic qualifications should have had several years' post-training experience in their trade and preferably have at least H.N.C. or equivalent qualifications.

Application forms from M.L.N.S., Technical and Scientific Register (K), 26 King Street, London, S.W.1, quoting D.558/5A/HS.

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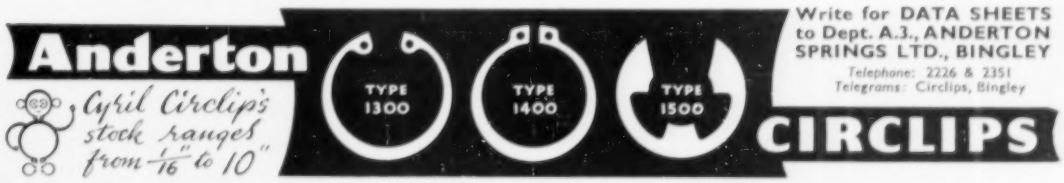

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Required for production of communication and radio apparatus. Also instrument makers, wirers and assemblers for factory test apparatus. Apply Personnel Manager, E. K. Cole, Ltd., Ekco Works, Malmesbury, Wilts.

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50	12	15	—	1 1/16	11/32	CE87B	2/9
100	12	15	—	1 1/16	1/2	CE100B	3/—
25	25	30	—	1 1/16	11/32	CE87C	2/9
50	25	30	—	1 1/16	11/32	CE88CE	3/—
100	25	30	—	1 1/16	1/2	CE100CE	4/—
12	50	60	—	1 1/16	11/32	CE87D	2/9
25	50	60	—	1 1/16	11/32	CE88DE	3/—
50	50	60	—	1 1/16	1/2	CE100DE	4/—
8	150	200	—	1 1/16	11/32	CE85FE	3/—
1	350	400	15	1 1/16	3/8	CE86L	2/6
2	350	400	25	1 1/16	11/32	CE88L	2/9
4	350	400	40	1 1/16	11/32	CE88LE	3/—
8	350	400	60	1 1/16	11/32	CE99LE	3/3
16	350	400	90	2 1/16	1/2	CE91LE	4/—
32	350	400	150	2 1/16	1 1/16	CE93LE	6/—
1	450	550	20	1 1/16	11/32	CE89P	2/9
2	450	550	30	1 1/16	11/32	CE89PE	3/—
4	450	550	40	1 1/16	11/32	CE99PE	3/3
8	450	550	60	1 1/16	1/2	CE90PE	3/6
16	450	550	100	1 1/16	1 1/16	CE92PE	5/—
32	450	550	160	2 1/16	1 1/16	CE94PE	7/6
40–40	150	200	—	2 1/16	1 1/16	CE98FE	7/6
32–32	275	325	—	2 1/16	1 1/16	CE95HE	8/6
8–8	450	550	80	2 1/16	1 1/16	CE98PEA	7/6
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Capacitance Microfarads	List Number	Dimensions L (inches)	D
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0.004	BM8	0.610	0.135
0.004	BM11	0.500	0.180
0.005	BM9	0.610	0.135
0.005	BM12	0.500	0.180
0.01	BM13	0.500	0.180
0.02	BM14	0.610	0.180
0.03	BM15	0.610	0.260
0.04	BM16	0.610	0.260
400 Volts D.C. Working			
0.0004	BM4	0.610	0.135
0.0005	BM5	0.610	0.135
0.001	BM6	0.610	0.135
0.002	BM18	0.500	0.180
0.003	BM19	0.500	0.180
0.005	BM20	0.610	0.180
0.01	BM21	0.610	0.260
600 Volts D.C. Working			
0.00005	BM25	0.500	0.180
0.0001	BM26	0.500	0.180
0.0001	BM1	0.610	0.135
0.0002	BM2	0.610	0.135
0.0002	BM27	0.500	0.180
0.00022	BM28	0.500	0.180
0.00025	BM29	0.500	0.180
0.0003	BM3	0.610	0.135
0.0003	BM30	0.500	0.180
0.0004	BM36	0.500	0.180
0.0005	BM31	0.500	0.180
0.001	BM32	0.500	0.180
0.002	BM33	0.610	0.260
0.003	BM34	0.610	0.260
0.004	BM35	0.610	0.260

If required, Type W97 can be supplied with a plastic insulating cover.

MIDGET 'THERMETIC' METALLISED PAPER

(HUNTS PATENT)

TEMPERATURE RANGE: —100 C to +100 C
Type approved to Joint Services Standard R.C.S. 136/A Category 40/100 Class H1

REGISTERED TRADE MARK

HUNTS CAPACITORS

THE TRADE MARK OF RELIABILITY

A. H. HUNT (Capacitors) LIMITED
WANDSWORTH · LONDON S.W.18
Telephone: BATtersea 1083

And in Canada:
HUNT CAPACITORS (Canada) LTD.
AJAX ONTARIO